Realistic Models of Superbursts in Accreting Neutron Stars

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Realistic Models of Superbursts
in Accreting Neutron Stars

A Dissertation Submitted to Kyushu University
for the Degree of Doctor of Science

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March 2004
Abstract

We investigate the phenomena called “X-ray bursts” observed by many X-ray satellites from more than 60 X-ray sources. The phenomena that the X-ray luminosity suddenly enhances by more than 100 times goes back to a quiescence phase in $10^{-100}$ s; they occur repeatedly for the every interval from a few hours to days. We confirm that the origin is considered to be thermonuclear burning runaways on an accreting neutron star.

Among all, we examine “superbursts” observed from 4U 1636-536. Though the “superburst” had a very long duration time of $\sim$ 6 hours, other feature such as the spectral evolution was similar to that of normal X-ray bursts. In particular, eight superbursts were observed from the seven X-ray sources, where 4U 1636-536 showed two superbursts, of which the interval was 4.7 years. At the present stage, there does not exist a model to explain quantitatively the origin of superbursts. On the other hand, we have wide consensus that X-ray bursts play an important role to restrict the thermal structure and the mass-radius relation of the neutron star. Therefore, we can consider that it is possible to construct more realistic models of the accreting neutron star by elucidating the mechanism of superbursts. We show that the superburst is due to thermonuclear burning ignited by accumulated fuels in the deep layers compared to normal X-ray bursts.

To obtain reasonable nuclear products during bursts, we perform the “post process” calculation, which operates a large nuclear reaction network to the results of evolutionary calculation with use of an approximation network. We carry out this post process calculation for successive two bursts, where we take into account the multi-zone effects and the effects of convection.

For the first time, we investigate three cases for the models of superbursts for in the calculations of X-ray burst simulations; helium flash, carbon flash for single burst and carbon flash accompanied with normal bursts. For helium flash, the burst shows the long duration while the accretion rate is different from the observation. Nevertheless, we suggest that the
helium flash could be one of the origins of the superbursts. For carbon flash, the burst
does not designate the long duration. However, since the temperature of the heated layer
becomes very high, the burning would become dynamical. Because our calculations assume
the hydrostatical equilibrium, the flash does not develop to a dynamical stage. Since the time
scale of the flash becomes comparable to that of a deflagration, we infer that the burning
leads to a long burst due to dynamical effects. For the case of carbon flash accompanied
with normal bursts, we carry out the calculations of successive 2786 normal bursts during
$1.81 \times 10^9$ s. We show the profiles of many normal bursts, light curves, temperature and
composition distributions. Finally we conclude that the carbon flash occurred in deep layers
after many normal bursts should trigger a superburst.
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1. Introduction

1.1. Observations and Theories of the X-Ray Bursts

Many low-mass X-ray binaries (LMXBs) show thermonuclear explosions, or so-called type I X-ray bursts (hereafter XRBs; Lewin et al. 1993). XRBs were discovered in 1975 independently by Grindlay & Gursky (1976) and Belian et al. (1976). Since then, XRBs have been observed from about 60 sources so far by the X-ray observatories, as Tenma, Ginga, the Rossi X-ray Timing Explorer (RXTE), Asca and Chandra. Such phenomena that the X-ray luminosity suddenly enhances by a factor of hundred are believed to be caused by the thermonuclear reaction on the surface of neutron stars. It is necessary to supply the fuel continuously, because XRBs occur repeatedly. Therefore, it is considered that neutron stars with XRBs should be accompanied by a donor star. A schematic image of LMXBs is shown in Fig. 1. Matter overflows from a companion star to a neutron star, forms an accretion disk and accretes on the surface of the neutron star. The rate of the matter accretion is called the accretion rate denoted by $\dot{M}$. Figure 2 shows galactic sky map observed by the Chandra, which is the X-ray satellite launched in 1999. LMXBs are indicated with the violet diamonds.

![Image of a low mass X-ray binary. A companion has a mass comparable to the solar mass ($M_\odot$). A primary star is a compact object such as a black hole, neutron star, or white dwarf. If XRB is observed, the primary can be identified as a neutron star.](image-url)
Fig. 2.— Galactic sky map of X-ray sources observed in Chandra (launched in 1999). http://chandra.harvard.edu/photo/map. Green: Galaxy, Active Galactic Nuclei and Quasar; Red: Galaxy Cluster; Yellow: Normal Star, White Dwarf and Star Cluster; Violet: Supernova Remnant, Neutron Star and Black Hole; Blue: Miscellaneous.

Profiles of XRBs are shown in Fig. 3 and the results of spectral analysis of the burst are shown in Fig. 4. A typical neutron star has the radius of 10 km and the mass of 1.4 $M_\odot$ (Lewin et al. 1993). In Fig. 4, the radius except for the expansion phase is smaller than 10 km. This is because there is a difference between the observed black-body temperature and the actual effective one, though both temperatures are taken to be identical in general. XRBs are observed first as rapid increase (1-10 s) in the X-ray flux, followed by exponential-like decline, where the typical duration is of the order of seconds to minutes. They recur with typical frequency of hours to days, which is considered to depend on the property of the accretion of fresh fuel. The luminosity of the burst, $L_b$, can also be fitted with the blackbody radiation (Swank et al. 1977):

$$L_b = S\sigma_{SB}T_{\text{eff}}^4$$  \hspace{1cm} (1)
Fig. 3.— X-ray burst profiles of 4U 1826-24 on 1998 June 24 in various energy bands of (RXTE/PCA). The decay time depends strongly on the X-ray photon energy. The interval of decay time is shorter at higher energies.

where $\sigma_{SB}$ and $T_{\text{eff}}$ are the Stefan-Boltzmann constant and the effective temperature, respectively. $S$ denotes the area which radiates X-ray flux. If the spherical symmetry is assumed, we have $S = 4\pi R^2$ with the radius of the black body $R$. In most bursts, $R$ stays nearly constant but $T_{\text{eff}}$ changes with time (see Fig. 4; Kong et al. 2003). $T_{\text{eff}}$ increases during the rising phase and decreases during the decay phase, which indicates the heating and subsequent cooling of the neutron star surface, respectively. XRBs observationally show the significant softening of the spectral feature (see Fig. 3). Then, this implies the cooling of the surface region due to the emission of the X-rays. Typical integrated burst energies are in the range of $10^{39} - 10^{40}$ ergs.

Many hot discussions had been done about the origin of the XRBs since the discovery in 1976. Finally, the essential features have been explained in terms of the unstable nuclear burning on accreting neutron stars—a thermonuclear runaway model (e.g. Joss 1978; Ayasli & Joss 1982; Wallace, Woosley, & Weaver 1982).

Properties of the XRBs are characterized by the following parameters, which are called
burst parameters (Hayakawa 1985; Kuulkers et al. 2002b):

\[
\alpha = \frac{\int F_{\text{pers}} \, dt}{\int F_b \, dt},
\]

(2)

\[
\gamma = \frac{F_{\text{pers}}}{F_b^{\text{peak}}},
\]

(3)

\[
\tau = \frac{\int F_b \, dt}{F_b^{\text{peak}}},
\]

(4)

where the observed flux is assumed to be the sum of the persistent flux \( F_{\text{pers}} \) and the burst flux \( F_b \); \( F_b^{\text{peak}} \) is the peak burst flux. These fluxes are defined as the energy passing through the unit area per unit time (ergs cm\(^{-2}\) s\(^{-1}\)). The two integrals in Eq. (2) are taken over the interval between the bursts for the \( F_{\text{pers}} \), and the burst phase for \( F_b \), respectively.

Based on the thermonuclear runaway model, we can express \( \alpha \) as the ratio of the grav-
itational energy release due to the accretion to the nuclear energy release during the burst:

\[ \alpha = \frac{GM\Delta M}{R} / (E_n\Delta M) \sim 10 - 10^2. \tag{5} \]

Here \( GM\Delta M/R \) is the gravitational potential energy of the accreting layer on the neutron star, \( E_n \) is the nuclear energy per unit mass, and \( \Delta M \) is the accumulated mass on the neutron star from a donor. The parameter \( \gamma \) should be a measure of the accretion rate. Especially, if the observed total flux during the burst becomes the Eddington luminosity (see (6)), \( \gamma \) approximately measures the accretion rate in units of the Eddington accretion rate because the Eddington luminosity is fixed for the neutron star. The duration time of the burst is given by \( \tau \), which is typically \( \sim 1 \) s and \( \sim 10 \) s for He burning and H burning, respectively. van Paradjis, Pennix, & Lewin (1988) have analyzed the bursts from the selected LMXBs, which show the photospheric radius expansion (PRE). It has been found that \(-2 \lesssim \log \gamma \lesssim -0.5\) where \( F_{\text{peak}}^b \) is set to be the peak flux that shows the PRE. Average values of \( \alpha \) and \( \tau \) in individual bursts for a given \( \gamma \) have been extracted.

Fujimoto (1985) has derived the relation \( R = 2.4 r_g \) for 4U 1636-536, taking account of the transverse Doppler effect due to the rapid rotation of the accreting matter just above the neutron star. In contrast, as plotted in Fig. 5, the mass-radius relation can be derived from a relation between the flux and color temperature in the bursts with the PRE (see the details in Fujimoto & Taam 1986; Ebisuzaki & Nomoto 1986).

Although many XRBs have been observed so far, there exist large uncertainties in the mass-radius relation for the neutron star. Therefore, fundamental physical processes should be clarified to determine the properties of the neutron stars; many faces of the neutron stars observed recently will be elucidated from some theoretical investigations, which would be also related to the many features of observations in the universe.
Fig. 5.— Mass–radius relation for the neutron star of 4U 1636-536 (Koike 2003). The relation derived by Fujimoto & Taam (1986) is denoted by the red solid curve. The ratio of the color to effective temperature is crucial to this analysis. In their paper, the ratio is assumed to be 1.4. The kilohertz quasi-periodic oscillation (kHz QPO) of 1.171 kHz was observed from this source. An allowed region derived from the analysis of the kHz QPO is inside the green dotted line in the mass-radius diagram (van der Klis 1997). We can see that large uncertainties still exist in the mass–radius relation.
Fig. 6.— X-ray burst light curve by RXTE in GX 3+1 at low (a), high (b) energies and the corresponding hardness curve (c), all with time resolution of 0.125 s. Time is set to be zero in 1999 August 10, that is, 18:35:53.3 UTC.

During some X-ray bursts the energy release is high enough that the luminosity at the surface of the neutron star reaches the Eddington limit. Then these bursts accompany the PRE (see Fig. 6). The light curve of the burst at low energy band is single-peaked, whereas at high energy band it is double-peaked (Fig. 6a, b). The corresponding hardness curve (Fig. 6c; the ratio of the count rates in high and low energy bands) shows that the burst first hardens, softens, hardens again, and then gradually softens again. In the meantime, the increase in the radius keeping the luminosity close to the Eddington limit leads to a temporary shift of the burst emission to lower energy photons within the X-ray band. This allows us to
evaluate whether the peak luminosity of the burst with PRE can be considered a standard candled (Van Paradijs 1978; Lewin 1984). If the PRE is very large, X-ray emissions between the precursor and the main peak disappear completely. We obtain the burst parameter $\alpha$ is $\sim 100$. Then it is considered that these bursts are due to the helium flash. Helium burning has the shorter rise time and duration than mixed hydrogen and helium burning have (see Fig. 7, Hashimoto et al. 2000).

\[ L_{Edd,\infty} = \frac{8\pi Gm_pMc[1 + (\alpha T_e)^{0.86}]}{\xi \sigma_T(1 + X)} \left( 1 - \frac{2GM}{R_{ph}c^2} \right)^{1/2}, \]  

For spherically symmetric emission, the Eddington luminosity measured by an observer at infinity is given by (Lewin et al. 1993)
where $m_p$ is the mass of proton, $c$ is the velocity of light, $\alpha_T$ is the coefficient parametrizing the temperature dependence of the electron scattering opacity ($\simeq 2.2 \times 10^{-9} \text{K}^{-1}$; Lewin et al. 1993), $X$ is the mass fraction of hydrogen in the atmosphere ($\approx 0.7$ for cosmic abundance), $R_{ph}$ is the radius of the photosphere, the parameter $\xi$ accounts for possible anisotropy of the burst emission and $\sigma_T$ is the Thomson cross section. The factor of parentheses in (6) represents the gravitational redshift due to the compact nature of the neutron star, which depends on the height of the emission region above the neutron star surface $R_{ph} \geq R$. Because $L_{Edd,\infty}$ depends on $R$ through the redshift factor, measurements of the peak flux for bursts with PRE allows in principle the measurement of these fundamental property (e.g. Kuulkers et al. 2002b). On the other hand, because the mass and radius of stable neutron stars predicted by any given equation of state span a narrow range, Eddington-limited X-ray bursts can be used as distance indicators (Galloway et al. 2003).

Recently, phenomena so-called superbursts have been detected from the seven X-ray bursters by BeppoSAX and RXTE; 4U 1735-44 (Cornelisse et al. 2000); 4U 1820-30 (Strohmayer 2000, Strohmayer & Brown 2001); KS 1731-260 (Kuulkers 2002); 4U 1636-536 (Wijnands 2001, Strohmayer & Markwardt 2002); Ser X-1 (Cornelisse et al. 2002); GX 3+1 (Kuulkers 2002) and 4U 1254-690 (in’t Zand et al. 2003). In particular, 4U 1636-536 produced two superbursts separated by 4.7 years (Wijnands 2001). Figure 8 (top) shows the superburst from KS 1731-260 as an example. Clearly, the light curve consists of a fast rise and slower exponential-like decay. During the rise up to the maximum in luminosity the spectrum hardens, whereas for the decay phase the spectrum softens (middle in Fig. 8). This is also reflected in the spectral fits to the time-resolved pre-burst subtracted from X-ray spectra (bottom in Fig. 8).

The observed properties of the superbursts observed are presented in Table 1. Each burst has energy of $10^{42}$ erg and duration of a few hours. They are usually best described by a black-body model. The effective temperature increases and decreases during the rise and decay phase, respectively. These superbursts are 1000 times luminous and 1000 times long.
Table 1: Properties of superbursts (Kuulkers et al. 2002a).

<table>
<thead>
<tr>
<th>Source</th>
<th>4U 1820-30</th>
<th>4U 1636-536</th>
<th>Ser X-1</th>
<th>4U 1735-44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td>PCA</td>
<td>PCA, ASM</td>
<td>WFC</td>
<td>WFC</td>
</tr>
<tr>
<td>Precursor</td>
<td>yes</td>
<td>yes</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Duration (hr)</td>
<td>&gt;2.5</td>
<td>~6</td>
<td>~4</td>
<td>~7</td>
</tr>
<tr>
<td>$\tau_{exp}$ (hr)</td>
<td>~1</td>
<td>1.05±0.01</td>
<td>1.2±0.1</td>
<td>1.4±0.1</td>
</tr>
<tr>
<td>$\tau_{rise}$ (min)$^a$</td>
<td>~2</td>
<td>~14</td>
<td>&lt;45</td>
<td>&lt;36</td>
</tr>
<tr>
<td>$kT_{max}$ (keV)</td>
<td>~3.0</td>
<td>2.35±0.01</td>
<td>2.6±0.2</td>
<td>2.6±0.2</td>
</tr>
<tr>
<td>$L_{peak}$ ($10^{38}$ erg sec$^{-1}$)$^{b,c}$</td>
<td>~3.4</td>
<td>~1.3</td>
<td>~1.6</td>
<td>~1.5</td>
</tr>
<tr>
<td>$E_b$ ($10^{42}$ erg)</td>
<td>≥1.4</td>
<td>~0.65</td>
<td>~0.8</td>
<td>≥0.5</td>
</tr>
<tr>
<td>$\tau \equiv E_b/L_{peak}$ (hr)$^c$</td>
<td>≥1.1</td>
<td>~1.4</td>
<td>~1.4</td>
<td>≥0.9</td>
</tr>
<tr>
<td>$L_{pers}$ ($L_{Edd}$)$^d$</td>
<td>~0.1</td>
<td>~0.1</td>
<td>~0.2</td>
<td>~0.25</td>
</tr>
<tr>
<td>$\gamma \equiv L_{pers}/L_{peak}$</td>
<td>~0.1</td>
<td>~0.3</td>
<td>~0.4</td>
<td>~0.4</td>
</tr>
<tr>
<td>$t_{noburst}$ (days)$^e$</td>
<td>&lt;167</td>
<td>&lt;41</td>
<td>~34</td>
<td>&gt;7.5</td>
</tr>
<tr>
<td>donor</td>
<td>He</td>
<td>H/He</td>
<td>?</td>
<td>H/He</td>
</tr>
</tbody>
</table>

$^a$Defined as the time between the peak of the precursor burst and the peak of the superburst.
$^b$Unabsorbed bolometric peak (black-body) luminosity.
$^c$The rise to the maximum is found in 4U 1820-30, 4U 1636-536 and 4U 1254-690; values of the others are to be used with caution.
$^d$The unabsorbed flux for the energy band of 0.01-100keV from spectral fits; the observed maximum flux during PRE bursts is used to define the Eddington luminosity.
$^e$Interval of cessation of normal type I X-ray bursts after the superburst.

in the duration compared with the normal bursts though the spectral evolution is similar.

RXTE was launched in 1995 December 30. This satellite carries three instruments, including the All-Sky Monitor (ASM), which consists of three Scanning Shadow Cameras (SSCs). The cameras are held stationary for 90 s intervals during which data are accumulated. Each 90 s interval provides intensities in the 1.5-12 keV band for all known sources in the field of view. The other two instruments are Proportional Counter Array (PCA). The PCA is an array of 5 co-aligned Proportional Counter Units and has 2-90 keV band pass and minimum 25 µm time resolution. BeppoSAX was launched on 1996 April 30. Onboard are two Wide Field Cameras (WFC). The band pass of the instruments is 2 to 28 keV.

Even now, there does not exist quantitative explanation of superbursts; the superbursts
Table 1: Continued.

<table>
<thead>
<tr>
<th>Source</th>
<th>GX 3+1</th>
<th>KS 1731-260</th>
<th>4U 1254-690</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td>ASM</td>
<td>WFC,ASM</td>
<td>WFC</td>
</tr>
<tr>
<td>Precursor</td>
<td>?</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Duration (hr)</td>
<td>&gt;3.3</td>
<td>∼12</td>
<td>∼14</td>
</tr>
<tr>
<td>$\tau_{exp}$ (hr)</td>
<td>1.6±0.2</td>
<td>2.7±0.1</td>
<td>6.0±0.3</td>
</tr>
<tr>
<td>$\tau_{rise}$ (min)$^a$</td>
<td>&lt;117</td>
<td>≃20</td>
<td>≤25</td>
</tr>
<tr>
<td>$kT_{max}$ (keV)</td>
<td>~2</td>
<td>2.4±0.1</td>
<td>1.8±0.1</td>
</tr>
<tr>
<td>$L_{peak}$ $(10^{38}$ erg sec$^{-1})^{b,c}$</td>
<td>~0.8</td>
<td>≃1.4</td>
<td>≃0.4</td>
</tr>
<tr>
<td>$E_b$ $(10^{42}$ erg)</td>
<td>≥0.6</td>
<td>≃1.0</td>
<td>≃0.8</td>
</tr>
<tr>
<td>$\tau \equiv E_b/L_{peak}$ (hr)$^c$</td>
<td>≥2.1</td>
<td>≃2.0</td>
<td>≃5.0</td>
</tr>
<tr>
<td>$L_{pers}$ $(L_{Edd})^{d}$</td>
<td>~0.2</td>
<td>≃0.1</td>
<td>≃0.13</td>
</tr>
<tr>
<td>$\gamma \equiv L_{pers}/L_{peak}$</td>
<td>~0.5</td>
<td>≃0.4</td>
<td>≃0.7</td>
</tr>
<tr>
<td>$t_{noburst}$ (days)$^e$</td>
<td>&lt;94</td>
<td>&gt;35</td>
<td>&lt;125</td>
</tr>
<tr>
<td>donor</td>
<td>?</td>
<td>?</td>
<td>H/He</td>
</tr>
</tbody>
</table>

last too long and their energy release is too much to be explained in terms of unstable burning of hydrogen/helium (Strohmayer & Brown 2001). Moreover, regular normal X-ray bursts are observed before the occurrence of the superburst (Kuulkers 2002) that include the precursor burst. The long rise and decay times of superbursts are consistent with the model of unstable burning in the deep layer below the hydrogen/helium burning layer. Therefore, it has been suggested that unstable burning of carbon is the origin of the superbursts (Cumming & Bildsten 2001, Strohmayer & Brown 2001).

If the accreted material onto the neutron star is pure helium, carbon can be produced when helium is burned either stably or unstably (Strohmayer & Brown 2001, Woosley et al. 2003). This would apply to the helium accretor 4U 1820-30 that shows long periods of high X-ray intensity during which no burst occurs, which is consistent with a period of stable helium burning. Note that unstable carbon burning only reproduce the observed feature in superbursts when we take into account neutrino losses and significant heat flux transported from deeper into the accreting layer of the neutron star (Strohmayer & Brown 2001). While

If the accreted material onto the neutron star is a mixture of hydrogen and helium, either unstable or stable burning of hydrogen/helium can produce carbon. However, the amount has been only limited (Koike 2003, Koike et al. 2004, Woosley et al. 2003). In those sources, normal XRBs have been observed with a mean rate of about 3 times per day during the period of the observation included each superburst (Cornelisse et al. 2002, Kuulkers et al. 2002a). This indicates that at least some amounts of the accreted material have burned stably before a superburst.

Cumming & Bildsten (2001) have shown that it is possible to ignite a small amount of carbon left after the hydrogen/helium burning, if carbon resides in a bath of heavy elements. These heavy elements are the products of unstable burning through the rp-process (see Appendix A) during the mixed hydrogen/helium XRBs (Schatz et al. 1999, Koike et al. 2004). In this case the superburst recurrence time would depend on accretion rates, being in the order of a few decades, a year to a decade, or a week to a month, according as the accretion rate is about 0.1, 0.3 or 1 times the Eddington accretion rate. More recently, it was suggested that the high temperature reached during a superburst induced the photodisintegration. This gives rise to an energy production comparable to that due to the carbon induced superburst (Schatz et al. 2003).

Another scenario was proposed by Kuulkers et al. (2002a). They suggested that hydrogen left after the burning in the hydrogen/helium layer is reignited by the electron capture that is followed by successive captures of neutrons by heavy nuclei occurred in the deeper layer. However, large amounts of hydrogen should be remained after bursts to explain the energy release in superbursts. Recent calculations have revealed that hydrogen is completely depleted after the hydrogen/helium burning (Koike 2003, Koike et al. 2004, Woosley et al. 2003). Nevertheless, the recurrence time of superbursts on the order of a year or less is to be expected (Kuulkers et al. 2002a, Kuulkers 2002).
Fig. 8.— *BeppoSAX*/WFC light curve (2-28 keV: top), hardness curve (ratio of the count rates in the 5-28 keV and 2-5 keV bands: middle). **Bottom:** effective black-body temperature derived from black-body model fits to the time resolved X-ray spectrum which is subtracted from the spectra during the superburst from KS 1731-260. (Kuulkers et al. 2002a)
1.2. Motivation in the Present Research

XRBs have been observed actively in recent years and among all a new type of burst as “superburst” has been found. Though many investigations on XRBs have been performed, there exist still many uncertainties in the explanation of the XRBs.

Woosley et al. (2003) proposed that the products of the second burst were different from that of the first burst. They used a one-dimensional hydrodynamical code and a large nuclear network ($Z = 1 - 84$). In the present paper, we investigate how the final products change during the recurrence of bursts. Using the evolutionary code of the neutron star (Fujimoto et al. 1984) and a large nuclear network (Koike 2003; $Z = 0 - 83$).

We construct realistic models of helium flash that are considered to be the origin of PRE bursts, using the evolutionary code. The sources with these bursts have the validity as distance indicators. The models of helium flash are significant for the study of XRBs. For superbursts we construct models for helium flash and carbon flash. Cumming and Bildsten (2001) suggested that a small amount of carbon ($^{12}\text{C} \approx 0.05 - 0.1$) is enough to trigger a thermonuclear runaway with energy comparable to the superburst. We construct the model of carbon flash based on the result of them. Especially, we construct the models with many repeated normal bursts toward the ignition of the carbon flash.

In §2 and 3, we present the evolutionary code of the neutron star and the one-zone model used in this paper. The condition about the ignition of the nuclear reaction runaway is indicated in §4 and the nuclear reaction networks used is presented in §5. In §6, we show the post process calculation, which is based on both the evolutionary calculation of the neutron star and the full nuclear reaction network; we compare our results with those of Koike et al. (2004). In §7, we present the models for helium and carbon flash, and discuss the results in detail in connection to superbursts.
2. Stellar Evolution Code of Neutron Star

The evolution code of a spherically symmetric neutron star (Hanawa & Fujimoto 1984; Fujimoto et al. 1984) is used in this investigation. The full set of general relativistic equations for the evolution of spherically symmetric stars (see Appendix B), as formulated by Thorne (1977), are solved by using a Henyey-type numerical scheme of implicit method (Sugimoto 1970). The star is divided into 266 meshes of the Lagrange mass-coordinate. The evolutionary changes in the temperature, density and chemical compositions are calculated at each mesh. The gravitational mass and radius of the neutron star are set to be $1.3 \, M_\odot$ and $8.1$ km, respectively. Then we have $\log g_s = 14.56$. The depth of the accretion layer is $\sim 100$ m and consists of 170 meshes (see Fig. 9). In the region of the combined hydrogen/helium burning, the interval between meshes is typically from $5 - 6$ cm.

![Fig. 9.— Schematic image of the neutron star adopted in the evolution model.](image)

In the accretion layer, the Eulerian coordinate is used, which is the most suitable method for computations of stellar structure when stellar mass varies (Sugimoto et al. 1981). We adopt independent variables, $t$ and the mass fraction $q$, which is defined as

$$q = \frac{M_r}{M}. \quad (7)$$

In general, $\partial/\partial t|_q$ is much smaller than $\partial/\partial t|_{M_r}$ in the outer layers, and consequently the computation time is greatly reduced. In this frame of reference, the equation of energy conservation is written as

$$L^*_t = L^{*nh}_{gr} + L^{*h}_{gr} + L^*_{nr} - L^*_{\nu r}, \quad (8)$$
\[ L_{gr}^{nh} = -M \int_0^q e^{\phi/c^2} \left[ T \left( \frac{\partial s}{\partial t} \right)_q + \mu_i \left( \frac{\partial N_i}{\partial t} \right)_q \right] dq, \tag{9} \]

\[ L_{gr}^h = \dot{M} \int_0^q e^{\phi/c^2} \left[ T \left( \frac{\partial s}{\partial \ln q} \right)_t + \mu_i \left( \frac{\partial N_i}{\partial \ln q} \right)_t \right] dq, \tag{10} \]

\[ L_{nr}^* = M \int_0^q e^{\phi/c^2} \varepsilon_n dq, \tag{11} \]

\[ L_{\nu r}^* = M \int_0^q e^{\phi/c^2} \varepsilon_\nu dq. \tag{12} \]

Here \( s, \mu_i, N_i, \dot{M}, \varepsilon_n \) and \( \varepsilon_\nu \) are the specific entropy, the chemical potential, the number of the \( i \)-th element per unit mass, the mass accretion rate, the nuclear energy generation rate per unit mass and the energy loss rate through neutrinos per unit mass, respectively. A factor \( e^{\phi/c^2} \) is the redshift correction factor for the gravitational potential \( \phi \). \( c \) is the speed of light. For \( M = 1.3M_\odot \) and \( R = 8 \text{km} \), \( e^{\phi/c^2} = 0.728 \). Symbols with asterisks indicate quantities measured by a distant observer; \( L_r \) and \( L_r^* \equiv e^{2\phi/c^2} L_r \) are the local luminosity and that observed far from the star, respectively. Eqs. (9) and (10) are called nonhomologous and homologous terms of the gravitational energy release, respectively.

We assume that material is accreted with the same entropy as that at the stellar surface, neglecting the surface effects caused by the accretion flow. Within the framework of the spherical symmetry, the kinetic energy of the falling material has little influence on the structure in the layer as deep as the burning shell, since the radial motion will be dissipated in the surface layers (Fujimoto et al. 1984).

For the inner part of the neutron star (\( \rho \geq 5 \times 10^7 \text{ g cm}^{-3} \)), our equation of state is based on the calculations by Baym, Pethick and Sutherland (1971); Negele (1974), Negele and Vautherin (1973); Baym, Bethe and Pethick (1971), and Pandharipande (quoted in Baym, Pethick and Sutherland 1971). For the outer part, an equation of state for an ideal gas plus radiation is taken into account, with the electron degeneracy and the Coulomb liquid correction by Slattery, Doolen and DeWitt (1979) taken into account. We present the mass-radius relation of the neutron star used in this investigation, \( M = 1.3M_\odot \) and \( R = 8 \text{km} \), as the red circle in Fig. 10. The green solid curve is the mass-radius relation obtained
Fig. 10.— Mass-radius relation for $g_s$ of the neutron star. The red circle on the green solid curve plots the size of the neutron star used in this investigation. The curve denotes the mass-radius relation deduced from the EOS by Baym, Pethick, & Sutherland (1971). Besides, the size of the typical neutron star, $M = 1.4M_\odot$ and $R=10\text{km}$, and 4U 1636-536 by the QPO observation are plotted by the orange box and the violet triangle, respectively.

from the EOS by Baym, Pethick and Sutherland (1971). For comparison, we show those of typical neutron star and 4U 1636-536 derived from the quasi-periodic oscillations (Kaaret et al. 1997), $M = 1.4M_\odot$ and $R=10\text{km}$, and $M = 2.0M_\odot$ and $R=9\text{km}$. Then, we obtain log $g_s = 14.38$ and 14.75 , respectively.

Neutrino emissivities include neutron-neutron and neutron-proton bremsstrahlung, modified Urca (Friman and Maxwell 1979), and electron-positron pair, photo and plasmon processes (Beaudet, Petrosian and Salpeter 1967). Opacities for materials of iron-rich mantle and neutron-rich core include the radiative opacities for $^{56}\text{Fe}$ calculated by Malone (1974), and the thermal conductivities calculated by Flowers and Itoh (1976) and Baym, Pethick
and Pines (1969). As to the opacities for lighter elements, $Z \leq 26$, we used the analytical approximations by Iben (1975), to the radiative opacities by Cox and Stewart (1970a, b), to the thermal conductivity for a nonrelativistic electron gas by Hubbard and Lampe (1969) and to the thermal conductivity for a relativistic electron gas by Canuto (1970). The criterion of the convection is the “Schwarzschild criterion” (see Appendix D).

Initial models have been constructed through the continuous accretion ($\dot{M} = \text{constant}$) without nuclear burning until the steady state is achieved, where the nonhomologous part (9) of the gravitational energy release vanishes (Fujimoto et al. 1984).
3. Shell Flash Model

The envelope of the neutron star is thin and approximated by a plane parallel configuration (Hanawa, Sugimoto, & Hashimoto 1983; Koike et al. 1999). The approximation is also adopted to the shell of the neutron star during bursts. The hydrostatic equation is integrated as

\[ P = \frac{\Delta M GM}{4\pi R^2} \frac{\mathcal{V}}{R^2} = g_s \sigma = 10^{22} g_{14} \sigma_8 \text{ dyn cm}^{-2}, \]

\[ g_s = \frac{GM}{R^2} \mathcal{V} = 1.3 \times 10^{14} \frac{M/M_\odot}{(R/10\text{km})^2} \text{ cm s}^{-2}, \]

\[ \sigma = \frac{\Delta M}{4\pi R^2} = 1.6 \times 10^{20} \frac{\Delta M/M_\odot}{(R/10\text{km})^2} \text{ g cm}^{-2}, \quad \mathcal{V} \equiv \left(1 - \frac{2GM}{Rc^2}\right)^{-1/2}, \]

where \( \sigma \) and \( g_s \) are the column density and the gravitational acceleration of the neutron star, respectively. \( g_{14} = g_s/10^{14} \text{ cm s}^{-2} \) and \( \sigma_8 = \sigma/10^8 \text{ g cm}^{-2} \) are the crucial parameters that are constant under the shell flash model. The general relativistic effect \( \mathcal{V} \) is taken into consideration for \( g_s \). From eq. (13), we know the relation between the ignition pressure and the mass of the accretion matter.

The equation of energy conservation, per unit mass, is expressed as

\[ c_p \frac{dT}{dt} = \epsilon_n - \epsilon_{\text{rad}}, \]

\[ \epsilon_{\text{rad}} = \frac{4acT^4}{3\kappa \sigma^2} \]

\[ = 1.5 \times 10^{17} T_9^4 (1 + 2.2 T_9) \left(\frac{\mu_e}{2}\right) \sigma_8^{-2} \text{ ergs g}^{-1} \text{ s}^{-1}. \]

The notations are the same as the preceding ones. Here, \( T \) and \( c_p \) are the temperature of the shell and the specific heat capacity under the constant pressure and \( \epsilon_{\text{rad}} \) is the radiative energy loss rate with the radiation constant \( a \). Since \( \epsilon_{\nu} \) is negligibly small in the regions discussed here, we can neglect it. Since the diffusion by the photon dominates the heat transfer in the synthesis of the heavy elements, we have set the radiative opacity in \( \epsilon_{\text{rad}} \) that includes the Compton effect such as \( \kappa = \kappa_{\text{Th}}/(1 + 2.2 T_9) \), where \( \kappa_{\text{Th}} \) and \( T_9 \) are the Thomson scattering opacity and the temperature in \( 10^9 \) K, respectively.
The total pressure consists of the contributions from ions, \( P_{\text{ion}} \), the partially degenerate electrons with relativistic electrons and positron in thermal equilibrium, \( P_e \), radiation, \( P_{\text{rad}} \), and the lattice Coulomb interaction, \( P_{\text{Coul}} \) (Hansen 1973). The density \( \rho \) can be computed from \( T \) and \( P \) with the aid of the equation of state \( P(\rho, T, \mu_e, \mu_I) \). The \( \mu_e \) and \( \mu_I \) are the molecular weight per electron and ion.

4. Condition of Ignition

The stability in thermal equilibrium configurations can be studied by introducing perturbations into eq. (15). Because the pressure remains constant during the burning, as seen in eq. (13), the perturbation equation is written as

\[
\frac{\partial \delta \ln T}{\partial t} = F \delta \ln T, \tag{16}
\]

\[
F \equiv \frac{\varepsilon_{\text{rad}}}{c_p T} \left[ \frac{\varepsilon_n}{\varepsilon_{\text{rad}}} \nu - 4 + \kappa_T + \left( \frac{\partial \ln \rho}{\partial \ln T} \right)_p \left( \frac{\varepsilon_n}{\varepsilon_{\text{rad}}} \xi + \kappa_\rho \right) \right], \tag{17}
\]

with

\[
\xi = \left( \frac{\partial \ln \varepsilon_n}{\partial \ln \rho} \right)_T, \quad \nu = \left( \frac{\partial \ln \varepsilon_n}{\partial \ln T} \right)_\rho, \]

\[
\kappa_\rho = \left( \frac{\partial \ln \kappa}{\partial \ln \rho} \right)_T, \quad \kappa_T = \left( \frac{\partial \ln \kappa}{\partial \ln T} \right)_\rho,
\]

where \( \delta \ln T \) is the perturbation in the temperature. The \( 3\alpha \) reaction depends strongly on the temperature and density compared with the \( \beta \) saturated Hot-CNO cycle. The energy generation rate, \( \varepsilon_{3\alpha} \), for the \( 3\alpha \) reaction is

\[
\varepsilon_{3\alpha} = 5.1 \times 10^{21} f \rho^2 Y^3 \exp(-44.04/T_8) \text{ erg g}^{-1} \text{ s}^{-1}, \tag{18}
\]

where \( f \) is the electron screening factor and \( Y \) is the mass fraction of He. Here we ignore the contribution of the non-resonant reaction. For \( \varepsilon_n = \varepsilon_{3\alpha} \), we get \( \xi \simeq 2 \) and \( \nu \simeq 44/T_8 - 3 \).

Fig. 11 shows \( T \)-sensitivity \( \nu \) for \( 3\alpha \), \( ^{14}\text{O}(\alpha,p)^{17}\text{F} \), and \( ^{15}\text{O}(\alpha,\gamma)^{19}\text{Ne} \) reactions. Since we set \( \delta \ln T \propto e^{FT} \), the nuclear burning in a thermal equilibrium is stable when \( F \) is negative.
Fig. 11.— Temperature sensitivity, \( \nu \), of the nuclear energy generation rate for the 3\( \alpha \) (solid line) the \( \alpha \)-induced reaction of \( ^{14}\text{O} \) (dotted line) and \( ^{15}\text{O} \) (dashed line). Screening factor is not included. (Koike 2003)

When \( F \) is positive, the burning is unstable and leads to a thermal runaway. Therefore, by solving \( F = 0 \) the ignition point of the shell flash can be obtained.

The ignition curve with \( F = 0 \) is plotted in Fig. 12 where the reaction rate by NACRE (Angulo et al. 1999) is used. The radiative and conductive opacities are taken from Iben (1975), Itoh et al. (1983), and Mitake et al. (1984). We classify the progress of nuclear burnings into three cases according to accretion rates (Fujimoto, Hanawa, & Miyaji 1981; Hanawa & Fujimoto 1982):

**Case 1** : Unstable helium flash in the hydrogen/helium mixture before the hydrogen depletion: \( \dot{M}(I) \leq \dot{M} \).

**Case 2** : Pure helium flash after the complete hydrogen depletion:
Fig. 12.— Progresses of shell burning for three cases. The ignition curves of the $3\alpha$ reaction for $Y = 1.0$, 0.1 is shown as red dot-dashed line and green dotted line. The violet dashed line is defined as the point where H decreases to 0.01 by mass fraction due to steady burning. The gravitational acceleration is taken as $\log g_s = 14.38$.

\[ \dot{M}(II) \leq \dot{M} \leq \dot{M}(I). \]

**Case 3**: Unstable hydrogen burning followed by the combined hydrogen/helium burning: $\dot{M} \leq \dot{M}(II)$.

Here $\dot{M}(I)$ and $\dot{M}(II)$ are the constant accretion rates during the stable burning to the points of I and II, respectively. Under the conditions of **Case 1** and **Case 3**, the rp-process should occur.

For the evolutionary code, the temperature evolves along the initial temperature distri-
bution until the ignition as seen in Fig. 32. The relation between $\dot{M}$ and the evolution of $T$ until the ignition is actually different from that between $\dot{M}$ and the initial temperature distribution, which is illustrated in detail in §7.
5. Nuclear Reaction Network for the rp-Process

5.1. Full Reaction Network

The isotopes contained in a old version of our network for the rp-process are limited to Kr \((Z = 36)\) as seen in Table 2 (Hashimoto & Arai 1985; Koike et al. 1999). Since the flow proceeds significantly beyond Kr (Schatz et al. 1998), we need the full nuclear reaction network (FNRN) that can include all the possible flows of the abundances. Therefore, Koike (2003) and Koike et al. (2004) have developed a nuclear reaction network up to Bi \((Z = 83)\) as seen in Table 3. Included channels are \((p, \gamma)\), \((\alpha, \gamma)\), \((\alpha, p)\), \((3\alpha, \gamma)\), \((n, \gamma)\), \((n, p)\), \((n, \alpha)\) and their inverse reactions; weak interactions; \(\beta^+\) decays and electron captures; delayed particle emissions if available.

Table 2: Nuclides incorporated in the reaction network used by Koike et al. (1999). \(A\) is the mass number. This network includes mainly the nuclear data by REACLIB (1995). See details in Koike et al. (1999).

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5.2. Nuclear Data and Energy Generation Rate

The nuclear data included in FNRN is taken from the following compilations:

1) Rauscher & Thielemann (2000, 2001; REACLIB).—Experimental and theoretical data (reaction rates, mass excesses, and partition functions).
Table 3: Nuclides contained in FNRN which is extended from the previous version of Table 2 (Koike et al. 2004).

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2) Angulo et al. (1999; NACRE).—Experimental reaction rates from H to Si.

3) Iliadis et al. (2001).—With (p,γ), (p,α) reaction rates from Ne to Ca.

4) Horiguchi, Tachibana, & Katakura (1996; Chart of the Nuclides).—β decays and particle emissions.


Reaction rates $^{56}\text{Ni}(p, \gamma)^{57}\text{Cu}(p, \gamma)^{58}\text{Zn}$ are adopted from Forstner et al. (2001). Though uncertain two proton capture processes in waiting points such as $^{68}\text{Se}(2p, \gamma)^{70}\text{Kr}$ could be important during the burst (Schatz et al. 1998), we use the data of REACLIB for $(p, \gamma)$ where the waiting points heavier than $^{64}\text{Ge}$ have negative $Q$-values. In the present calculations, the abundance flows proceed beyond the waiting points through the successive $(p, \gamma)$ reactions and/or $\beta^+$ decays after $(p, \gamma)$. Therefore, we can say that the inclusion of $(2p, \gamma)$ reactions does not change our results significantly.

Considering the importance in high density, we include the electron screening factors; weak and intermediate screening factors are taken from Graboske et al. (1973) and the strong screenings are from Itoh et al. (1979).

The rate equation to calculate the composition change is solved by means of the iterative method (Hashimoto & Arai 1985). The equation to determine the abundance change in time is

$$\frac{dY_i}{dt} = N^i_j \lambda_j Y_j + N^i_{j,k} \rho N_A \langle j, k \rangle Y_j Y_k + N^i_{j,k,l} \rho^2 N^2_A \langle j, k, l \rangle Y_j Y_k Y_l. \quad (19)$$

We note that the same scripts of $i, j, k$ indicate to take the summation. $Y_i$ is the abundance of the $i$-th particle, which is related to the mass fraction $X_i$ and the atomic mass $A_i$ in amu by $Y_i = X_i/A_i$. $N^{N-1}_A \langle j, k, \ldots \rangle$ is the thermally averaged reaction rate, which has the dimension of $(\text{cm}^3 \text{ mol}^{-1})^{N-1} \text{ s}^{-1}$ for the N-body reaction. $N_A$ is the Avogadro number. The number density of the $i$-th particle is given by $\rho N_A Y_i$. The coefficients are defined by $N^i_j = N_i$, $N^i_{j,k} = N_i/(N_j!N_k!)$, and $N^i_{j,k,l} = N_i/(N_j!N_k!N_l!)$ where $N_i$ represents the positive (negative) number how many the $i$th-particles are created (destroyed) in the reaction, and the factorials of $N_i$ avoid the multiple counting of reactions when identical particles react each other.

Using the abundance difference for $dt$, the nuclear energy generation rate $\varepsilon_n$ can be obtained with the binding energy of the $i$-th particle $B_i$ in units of MeV which is taken from
Audi & Wapstra (1995):

\[ \varepsilon_n = 9.6485 \times 10^{17} \sum_i \frac{dY_i}{dt} B_i \text{ erg g}^{-1} \text{ s}^{-1}. \]  
(20)

5.3. Approximation Reaction Network

5.3.1. Approximation Reaction Network for Combined Hydrogen/Helium Burning

If we couple the large reaction network presented in Table 3 with the stellar evolution code in §2, it takes very long computational time to carry out the evolutionary calculation. Therefore, to save the computing time in XRB simulations, the approximation reaction network (APRN1) has been devised (Wallace & Woosley 1981; Hanawa, Sugimoto, & Hashimoto 1983; Rembges et al. 1997). In this thesis, we use the APRN1 that includes 16 nuclides: \(^1\text{H}, \quad ^4\text{He}, \quad ^{12}\text{C}, \quad ^{14}\text{O}, \quad ^{15}\text{O}, \quad ^{16}\text{O}, \quad ^{17}\text{F}, \quad ^{22}\text{Mg}, \quad ^{30}\text{S}, \quad ^{56}\text{Ni}, \quad ^{60}\text{Ni}, \quad ^{60}\text{Zn}, \quad ^{64}\text{Ge}, \quad ^{68}\text{Ge}, \quad \text{and} \quad ^{68}\text{Se}\) with updated nuclear reaction rates (Hanawa et al. 1983). The nuclear chart and typical abundance flows in the rp-process for the APRN1 are shown in Fig. 13. In the APRN1, only 34 reactions are considered for composition changes and energy generation. The steady-state conditions, which can be generally applied during the operation of rp-process, allow a single reaction to represent frequently an entire reaction chain (where the slowest reaction in the reaction sequences is usually employed to govern the rates at which the sequences operate).

The conversion rate \(\Lambda_1\) from \(^{22}\text{Mg}\) to \(^{30}\text{S}\) (the orange dotted line in Fig. 13) depends on the thermodynamic conditions except for the case of the neutron star. Though the reaction flow waits at \(^{22}\text{Mg}\) due to \(\beta^+\) decay, it continues to \(^{30}\text{S}\) for the rp-process. The higher density encountered in the accreting neutron star allows \(^{22}\text{Mg}(p, \gamma)^{23}\text{Al}\) to proceed faster than \(^{22}\text{Mg}(e^+, \nu)^{22}\text{Na}\), resulting in a slightly different path. As a result, the sum of the mean \(\beta^+\) decay lifetime, normally 9.3 s, is reduced to only 0.95 s for the characteristic flow path in the nuclear flash on a neutron star. At higher temperature and density, \((\alpha, p)\) reactions
Fig. 13.— Approximation reaction network for hydrogen/helium burning (APRN1). The arrows denote abundance flows for nuclear reactions that occur in a typical rp-process.

begin to bridge the $\beta^+$ decays at $^{22}\text{Mg}$ and $^{26}\text{Si}$;

$$
\Lambda_1 = \max \left\{ \frac{1.05}{\rho y_\alpha \lambda_{\alpha p}(^{26}\text{Si})}, \right\}
$$

where 1.05 is the sum of the mean lifetimes of $\beta^+$ decay from $^{22}\text{Mg}$ to $^{30}\text{S}$ during a flash, $y_\alpha$ is the helium abundance and $\lambda_{\alpha p}(^{26}\text{Si})$ is the rate of $^{26}\text{Si}(\alpha,p)^{29}\text{P}$ reaction. The conversion rate $\Lambda_2$ from $^{30}\text{S}$ to $^{56}\text{Ni}$ (orange solid line in Fig. 13) is as follows:

$$
\Lambda_2 = \max \left\{ \frac{0.067}{\rho y_\alpha \lambda_{\alpha p}(^{44}\text{Ti})}, \right\}
$$

where 0.067 is the sum of mean lifetimes through a series of ($\alpha,p$) and $\beta^+$ decay reactions from $^{30}\text{S}$ to $^{56}\text{Ni}$. The $\beta^+$ decay rate of unstable nuclides $^{64}\text{Ge}$ and $^{68}\text{Se}$ are $1.083 \times 10^{-2}$ and $7.220 \times 10^{-3}$. APRN1 is constructed to reproduce approximately the nuclear energy generation rate calculated by the large network up to Kr with use of the shell flash model (see Fig. 14). In Fig. 14, we set the time to be zero when $\varepsilon_n$ is $2.8 \times 10^{14}$ erg g$^{-1}$ s$^{-1}$, which
Fig. 14.— Nuclear energy generation rate by the FNRN (green line) and the APRN1 (red line) with use of the shell flash model. The parameters are log \( P = 23.0 \) and \( \log g_s = 14.75 \). The initial temperature is set to be \( 1.5 \times 10^8 \) K and compositions are H (73.0%), \(^{4}\text{He}\) (25.0%), \(^{14}\text{O}\) (0.7%), and \(^{15}\text{O}\) (1.3%).

is \( 2\varepsilon_{\text{Hot–CNO}} \) (see (A1) in Appendix A). The time integrations from -50 to 500 s in Fig. 14 for the FNRN and the APRN1 are \( 4.27 \times 10^{18} \) and \( 4.16 \times 10^{18} \) erg g\(^{-1}\), respectively.
5.3.2. **Approximation Reaction Network for Helium Burning**

We make the approximation reaction network for helium burning (APRN2) to study the flashes which cause bursts with PRE. APRN2 includes 13 alpha particles: $^4\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$, $^{20}\text{Ne}$, $^{24}\text{Mg}$, $^{28}\text{Si}$, $^{32}\text{S}$, $^{36}\text{Ar}$, $^{40}\text{Ca}$, $^{44}\text{Ti}$, $^{48}\text{Cr}$, $^{52}\text{Fe}$ and $^{56}\text{Ni}$, and contains $(3\alpha, \gamma)$, $(\alpha, \gamma)$, $(\alpha, p)$ reactions and their inverse ones (Fig. 15). The data of NACRE (Angulo et al. 1999) is used for the $3\alpha$ reaction rate and those of REACLIB for the others. We assume that the $(p, \gamma)$ reaction follows instantaneously the $(\alpha, p)$ reaction.

Fig. 15.— Chart of the approximation reaction network for helium burning (APRN2). As shown in Fig. 13, the arrows denote the abundance flows in nuclear reactions.
6. Post Process Calculation

The accretion layers during the bursts are well approximated to have flat configurations during the flash unless the luminosity exceeds the Eddington limit because of the large gravitational potential (Fujimoto, Hanawa, & Miyaji 1981; Hanawa & Fujimoto 1982). Since the pressure in the burning layer is determined from the weight of the accumulated layers, ignition pressure should be the critical parameter for the XRB model. Based on this picture, Koike et al. (2004) investigated the relation between the ignition pressure and the final products, with use of the shell flash model and the FNRN (Table 3: $Z = 0 – 83$).

Using the one-dimensional implicit hydrodynamic code and full nuclear reaction network ($Z = 1 – 84$), Woosley et al. (2003) carried out the calculation of successive bursts. As the result, they proposed that there exist significant differences between products of the first burst and the following bursts. Therefore, we advance the calculation by Koike et al. (2004) and we carry out nucleosynthesis simulation for multiple Euler meshes with including the effects of the convection.

6.1. Procedure

We perform 'post-process' calculations (PPCs) which require a realistic neutron star model constructed with use of the spherically symmetric evolutionary code (Hanawa & Fujimoto 1984; Fujimoto et al. 1984; §3). To get the density and temperature history as a function of time, we simulate XRBs with use of the evolutionary code for $\dot{M} = 3.0 \times 10^{-9}$ and $3.0 \times 10^{-10} \, M_\odot \, yr^{-1}$. We adopt the nuclear reaction network of APRN1, and the initial compositions in accretion matter are assumed to be H (73.0%), $^4$He (25.0%), $^{14}$O (0.7%) and $^{15}$O (1.3%). Figure 16 shows two successive bursts for $\dot{M} = 3.0 \times 10^{-9} \, M_\odot \, yr^{-1}$. The profiles of pressure, density and temperature are shown in Fig. 17. Figure 18 shows the density and temperature profiles of the mesh where the temperature reaches the maximum among the layers during “burst 1” and “burst 2”. The case of $3.0 \times 10^{-10} \, M_\odot \, yr^{-1}$ is shown in Fig. 20.
Using the FNRN, we start the PPCs from the composition obtained by the APRN1 just before “burst 1” (see Fig. 19). Then we consider the change in composition through the convection, which include the mixing of the final products of “burst 1” and hydrogen/helium just before “burst 2”. This is the composition before “burst 2”, with which we restart the PPCs.
Fig. 17.— Pressure-density-temperature profile during “burst 1” for $3.0 \times 10^{-9} \, M_{\odot} \, \text{yr}^{-1}$ with $\log P = 22.5 - 23$. Each line corresponds to each mesh in accretion layers.

Fig. 18.— Relation between density and temperature of the mesh which undergoes the maximum temperature among the layers during the “burst 1” (red line) and “burst 2” (green line). The arrows represent the direction of the time elapsed.
Fig. 19.— Composition distribution vs. pressure by the APRN1 just before the “burst 1” for $3 \times 10^{-9} M_\odot$ yr$^{-1}$.

Fig. 20.— Same as Fig. 18 except for $3 \times 10^{-10} M_\odot$ yr$^{-1}$. 
6.2. Results

Fig. 21.— Abundance flows on the chart of nuclides for $3 \times 10^{-9} M_\odot$ yr$^{-1}$. From the upper left figure, the elapsed times are $1.5 \times 10^{-5}$ s, $6.0 \times 10^{-1}$ s, 1.1 s, 1.2 s, 1.7 s and $1.3 \times 10^1$ s.

Figures 21, 22 and 23 show the abundance flows described on the chart of nuclides at the mesh that undergoes the maximum temperature among the layers during “burst 1” for $3 \times 10^{-9} M_\odot$ yr$^{-1}$. The case of $3 \times 10^{-10} M_\odot$ yr$^{-1}$ is shown in Figs. 24, 25 and 26. The final products of “burst 1” and “burst 2” as well as the results of Koike et al. (2004) are presented in Tables 4 and 5 for $3 \times 10^{-9}$ and $3 \times 10^{-10} M_\odot$ yr$^{-1}$, respectively. In these calculations, H is
Fig. 22.— Same as Fig. 21 but that the elapsed times are $5.2 \times 10^1$ s, $2.1 \times 10^2$ s, $2.7 \times 10^2$ s and $1.4 \times 10^3$ s.

found to be exhausted. The products in “burst 1” are not so different from those of Koike et al. (2004), though their calculation did not include the convection. The lighter elements exist more abundant in “burst 2” than in “burst 1”.
Table 4: Mass fractions of abundant nuclei in “burst 1” and “burst 2” together with the results of Koike et al. (2004) for $3 \times 10^{-9} M_\odot \text{ yr}^{-1}$. The abundances are evaluated at the mesh of the maximum temperature.

<table>
<thead>
<tr>
<th></th>
<th>burst 1</th>
<th>burst 2</th>
<th>Koike et al. (2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^{64}\text{Zn}$ 3.16(-1)</td>
<td>$^{66}\text{Ni}$ 4.70(-1)</td>
<td>$^{64}\text{Zn}$ 2.65(-1)</td>
</tr>
<tr>
<td>2</td>
<td>$^{68}\text{Ge}$ 1.85(-1)</td>
<td>$^{48}\text{Ca}$ 9.40(-2)</td>
<td>$^{68}\text{Ge}$ 1.69(-1)</td>
</tr>
<tr>
<td>3</td>
<td>$^{72}\text{Se}$ 1.38(-1)</td>
<td>$^{71}\text{Ga}$ 4.69(-2)</td>
<td>$^{72}\text{Se}$ 1.37(-1)</td>
</tr>
<tr>
<td>4</td>
<td>$^{76}\text{Kr}$ 7.74(-2)</td>
<td>$^{47}\text{Ca}$ 4.20(-2)</td>
<td>$^{76}\text{Kr}$ 7.79(-2)</td>
</tr>
<tr>
<td>5</td>
<td>$^{81}\text{Rb}$ 4.52(-2)</td>
<td>$^{20}\text{Ne}$ 2.54(-2)</td>
<td>$^{81}\text{Rb}$ 4.08(-2)</td>
</tr>
<tr>
<td>6</td>
<td>$^{77}\text{Kr}$ 1.13(-2)</td>
<td>$^{36}\text{S}$ 2.46(-2)</td>
<td>$^{81}\text{Rb}$ 2.01(-2)</td>
</tr>
<tr>
<td>7</td>
<td>$^{4}\text{He}$ 1.10(-2)</td>
<td>$^{65}\text{Cu}$ 2.27(-2)</td>
<td>$^{68}\text{As}$ 1.83(-2)</td>
</tr>
<tr>
<td>8</td>
<td>$^{82}\text{Sr}$ 1.05(-2)</td>
<td>$^{46}\text{Ca}$ 2.19(-2)</td>
<td>$^{80}\text{Sr}$ 1.14(-2)</td>
</tr>
<tr>
<td>9</td>
<td>$^{80}\text{Sr}$ 1.04(-2)</td>
<td>$^{40}\text{Ar}$ 1.76(-2)</td>
<td>$^{4}\text{He}$ 1.10(-2)</td>
</tr>
<tr>
<td>10</td>
<td>$^{83}\text{Sr}$ 9.42(-3)</td>
<td>$^{16}\text{O}$ 1.60(-2)</td>
<td>$^{82}\text{Sr}$ 1.01(-2)</td>
</tr>
<tr>
<td>11</td>
<td>$^{12}\text{C}$ 7.46(-3)</td>
<td>$^{36}\text{Cl}$ 1.46(-2)</td>
<td>$^{77}\text{Kr}$ 1.01(-2)</td>
</tr>
<tr>
<td>12</td>
<td>$^{90}\text{Mo}$ 6.61(-3)</td>
<td>$^{38}\text{Ar}$ 1.39(-2)</td>
<td>$^{93}\text{Tc}$ 7.81(-3)</td>
</tr>
<tr>
<td>13</td>
<td>$^{93}\text{Tc}$ 6.48(-3)</td>
<td>$^{44}\text{Ca}$ 6.82(-3)</td>
<td>$^{12}\text{C}$ 7.65(-3)</td>
</tr>
<tr>
<td>14</td>
<td>$^{86}\text{Zr}$ 6.39(-3)</td>
<td>$^{41}\text{Ar}$ 6.75(-3)</td>
<td>$^{90}\text{Mo}$ 7.11(-3)</td>
</tr>
<tr>
<td>15</td>
<td>$^{85}\text{Y}$ 5.91(-3)</td>
<td>$^{24}\text{Mg}$ 6.26(-3)</td>
<td>$^{86}\text{Zr}$ 6.36(-3)</td>
</tr>
<tr>
<td>16</td>
<td>$^{73}\text{Se}$ 5.43(-3)</td>
<td>$^{66}\text{Zn}$ 6.18(-3)</td>
<td>$^{83}\text{Sr}$ 5.96(-3)</td>
</tr>
<tr>
<td>17</td>
<td>$^{94}\text{Ru}$ 3.92(-3)</td>
<td>$^{42}\text{Ar}$ 6.04(-3)</td>
<td>$^{94}\text{Ru}$ 5.32(-3)</td>
</tr>
<tr>
<td>18</td>
<td>$^{95}\text{Ru}$ 3.83(-3)</td>
<td>$^{25}\text{Mg}$ 6.02(-3)</td>
<td>$^{83}\text{Y}$ 4.45(-3)</td>
</tr>
<tr>
<td>19</td>
<td>$^{89}\text{Nb}$ 3.68(-3)</td>
<td>$^{63}\text{Ni}$ 5.76(-3)</td>
<td>$^{73}\text{Se}$ 4.35(-3)</td>
</tr>
<tr>
<td>20</td>
<td>$^{89}\text{Nb}$ 3.68(-3)</td>
<td>$^{63}\text{Ni}$ 5.76(-3)</td>
<td>$^{73}\text{Se}$ 4.35(-3)</td>
</tr>
</tbody>
</table>
Fig. 23.— Enlarged flows for \( t = 1.5 \times 10^{-5} \) and \( 1.3 \times 10^1 \) s in Fig. 21.
Fig. 24.— Same as Fig. 21 except for $3 \times 10^{-10} M_\odot$ yr$^{-1}$. The elapsed times are $1.0 \times 10^{-5}$ s, $4.1 \times 10^{-1}$ s, $4.8 \times 10^{-1}$ s, $5.2 \times 10^{-1}$ s, $5.7 \times 10^{-1}$ s, 1.2 s, 1.4 s and 2.0 s.
Fig. 25.— Same as Fig. 24 but that the elapsed times are 4.6 s and 9.9×10^2 s

Table 5: Same as Table 4 but for 3×10^{-10}M_⊙ yr^{-1}.

<table>
<thead>
<tr>
<th></th>
<th>burst 1</th>
<th>burst 2</th>
<th>Koike et al. (2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64Zn 2.54(-1)</td>
<td>36Ar 3.16(-1)</td>
<td>64Zn 3.47(-1)</td>
</tr>
<tr>
<td>2</td>
<td>56Ni 1.64(-1)</td>
<td>32S 1.08(-1)</td>
<td>56Ni 1.85(-1)</td>
</tr>
<tr>
<td>3</td>
<td>60Ni 1.01(-1)</td>
<td>39K 1.02(-1)</td>
<td>64Ga 8.23(-2)</td>
</tr>
<tr>
<td>4</td>
<td>12C 6.10(-2)</td>
<td>64Zn 9.33(-2)</td>
<td>60Ni 6.39(-2)</td>
</tr>
<tr>
<td>5</td>
<td>39K 4.90(-2)</td>
<td>40Ca 7.63(-2)</td>
<td>12C 3.65(-2)</td>
</tr>
<tr>
<td>6</td>
<td>32S 4.18(-2)</td>
<td>12C 6.66(-2)</td>
<td>55Co 3.44(-2)</td>
</tr>
<tr>
<td>7</td>
<td>4He 3.51(-2)</td>
<td>56Ni 6.05(-2)</td>
<td>32S 3.43(-2)</td>
</tr>
<tr>
<td>8</td>
<td>38Ar 3.33(-2)</td>
<td>4He 4.83(-2)</td>
<td>39K 3.18(-2)</td>
</tr>
<tr>
<td>9</td>
<td>55Co 3.05(-2)</td>
<td>60Ni 3.70(-2)</td>
<td>4He 2.24(-2)</td>
</tr>
<tr>
<td>10</td>
<td>34S 2.73(-2)</td>
<td>55Co 1.06(-2)</td>
<td>36Ar 2.19(-2)</td>
</tr>
<tr>
<td>11</td>
<td>35Cl 2.23(-2)</td>
<td>56Co 9.73(-3)</td>
<td>54Fe 1.42(-2)</td>
</tr>
<tr>
<td>12</td>
<td>56Co 1.94(-2)</td>
<td>35Cl 8.44(-3)</td>
<td>68Ge 1.37(-2)</td>
</tr>
<tr>
<td>13</td>
<td>54Fe 1.69(-2)</td>
<td>28Si 6.55(-3)</td>
<td>35Cl 1.30(-2)</td>
</tr>
<tr>
<td>14</td>
<td>50Cr 1.56(-2)</td>
<td>54Fe 5.83(-3)</td>
<td>38Ar 1.29(-2)</td>
</tr>
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<td>36Ar 1.44(-2)</td>
<td>65Zn 4.59(-3)</td>
<td>50Cr 1.22(-2)</td>
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<td>40Ca 1.07(-3)</td>
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<td>28Si 1.09(-2)</td>
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<td>28Si 6.27(-3)</td>
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<td>33S 7.98(-3)</td>
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<td>60Cu 5.89(-3)</td>
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<td>40Ca 7.96(-3)</td>
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<td>51Mn 5.31(-3)</td>
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<tr>
<td>20</td>
<td>42Ca 7.50(-3)</td>
<td>51Cr 2.56(-3)</td>
<td>43Sc 4.20(-3)</td>
</tr>
</tbody>
</table>
Fig. 26.— Enlarged flows for $t = 1.0 \times 10^{-5}$ and 2.0 s in Fig. 24.
Woosley et al. (2003) presented the multi-zone models of XRBs using an adaptive nuclear reaction network up to Po ($Z = 84$). The mass and radius of the neutron star which they used were $1.4M_\odot$ and 10km. They calculated four cases of two accretion rates, $3.5 \times 10^{-10}$ and $1.75 \times 10^{-9}M_\odot$ yr$^{-1}$, and two initial abundances, H (70.48%), $^4$He (27.52%) and $^{14}$N (2%); $Z = Z_\odot$, and H (75.9%), $^4$He (24%) and $^{14}$N (0.1%); $Z = 0.05Z_\odot$. Figures 27 and 28 show the light curves of the second burst for our calculation ($3 \times 10^{-10}M_\odot$ yr$^{-1}$) and Woosley et al. (2003) ($3.5 \times 10^{-10}M_\odot$ yr$^{-1}$ and $Z_\odot$), respectively. Global shape of the light curve resembles well each other, though our duration is shorter than theirs. Table 6 gives the mass fractions of abundant nuclei, which was read from the figure of the composition distribution in Woosley et al. (2003) for $3.5 \times 10^{-10}M_\odot$ yr$^{-1}$ and $Z_\odot$. We can see that the yields of our model are heavier than theirs, and the duration time of the burst is shorter. These differences could be ascribed to the following treatment:

1) Convection: we adopted the Schwarzschild criterion and assume the instantaneous mixing, while they used the Ledoux criterion (see Appendix D) and a semi-convective diffusion equation was solved for mixing.

2) Neutron star model: we construct a realistic model of a neutron star, while they take account only the accretion layer under a plausible boundary condition.

3) Initial abundance: this causes different thermal structure through the opacity for accreting layers.

4) Accretion rate: small differences of $\dot{M}$ induce significant thermal effects on the accretion layer.

<table>
<thead>
<tr>
<th></th>
<th>burst 2</th>
<th>Woosley et al. (2003)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$^{36}$Ar 3.16(-1)</td>
<td>$^{28}$Si 2.6(-1)</td>
</tr>
<tr>
<td>2</td>
<td>$^{32}$S 1.08(-1)</td>
<td>$^{40}$Ca 2.3(-1)</td>
</tr>
<tr>
<td>3</td>
<td>$^{39}$K 1.02(-1)</td>
<td>$^{24}$Mg 1.1(-1)</td>
</tr>
<tr>
<td>4</td>
<td>$^{64}$Zn 9.33(-2)</td>
<td>$^{12}$C 4.0(-2)</td>
</tr>
<tr>
<td>5</td>
<td>$^{40}$Ca 7.63(-2)</td>
<td>$^{30}$K 9.5(-3)</td>
</tr>
</tbody>
</table>

Table 6: Mass fractions of abundant nuclei at the second burst of $3 \times 10^{-10}M_\odot$ yr$^{-1}$ and those by Woosley et al. (2003) of $3.5 \times 10^{-10}M_\odot$ yr$^{-1}$. 
Fig. 27.— Light curve of “burst 2” with $3\times10^{-10} M_\odot$ yr$^{-1}$.

Fig. 28.— Light curve of the second burst with $3.5\times10^{-10} M_\odot$ yr$^{-1}$ and $Z_\odot$ in Woosley et al. (2003).
7. Realistic Models of Accreting Neutron Star

In this section, we develop the models of normal bursts and superbursts using the evolutionary code which include the approximation network (APRX1 or APRX2). Especially, we concentrate normal bursts of the helium flash, which is considered to be the origin of bursts accompanied with PRE. For superburst, we investigate the three models; helium flash, carbon flash with a single burst, and carbon flash accompanied with many normal bursts.

7.1. Bursts with the Pure Helium Accretion

We simulate helium flashes using the evolutionary code explained in §2 with APRN2 (Fig. 15). For accretion rates ($\dot{M}$), we select three cases of $3 \times 10^{-8} (\approx M_{\text{Edd}})$, $3 \times 10^{-9}$, and $3 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$. The accreting matter is assumed to be pure $^4\text{He}$: $Y = 1.0$. Burst parameters (van Paradijs et al. 1988) obtained from our calculation are listed in Table 7, and the time evolution of the luminosity for each $\dot{M}$ is shown in Fig. 29. The dotted, solid and dashed lines are $3 \times 10^{-8}$, $3 \times 10^{-9}$ and $3 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$, respectively.

Table 7: Burst parameters for three accretion models.

<table>
<thead>
<tr>
<th>$\dot{M}$ ($M_{\odot} \text{ yr}^{-1}$)</th>
<th>$3 \times 10^{-10}$</th>
<th>$3 \times 10^{-9}$</th>
<th>$3 \times 10^{-8}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{\text{rise}}$ (s)</td>
<td>$6.3 \times 10^2$</td>
<td>36</td>
<td>1.1</td>
</tr>
<tr>
<td>$L_{\text{peak}}$ ($L_{\odot} \text{ erg s}^{-1}$)</td>
<td>$3.9 \times 10^4$</td>
<td>$6.9 \times 10^4$</td>
<td>$6.6 \times 10^4$</td>
</tr>
<tr>
<td>$t_{e-folding}$ (s)</td>
<td>$4.3 \times 10^3$</td>
<td>$1.8 \times 10^2$</td>
<td>4.3</td>
</tr>
</tbody>
</table>

The burst with the lower accretion rate results in the longer duration and is more energetic because the ignition occurs at the deeper region of the accreted layer where a lot of nuclear fuels have been stored. The duration of the burst for $\dot{M} = 3 \times 10^{-10} M_{\odot} \text{ yr}^{-1}$ is more than 3 hours. Although the heat produced in the layers of $P \geq 10^{26} \text{ dyn cm}^{-2}$ conducted significantly to the inner layers, the total energy of the burst has exceeded $10^{42}$ ergs.
Fig. 29.— Light curves for $\dot{M} = 3 \times 10^{-8}$ (green dotted line), $3 \times 10^{-9}$ (red solid line) and $3 \times 10^{-10} M_\odot \, \text{yr}^{-1}$ (blue dashed line). Time is set to be zero at the beginning of each burst.

**7.2. Comparison with the Observation of 4U 1820-30**

The X-ray source 4U 1820-30 in the globular cluster NGC 6624 was first identified as the X-ray burster. It has been suggested that the donor star is a helium dwarf with a mass of 0.06–0.08 $M_\odot$ (Rappaport et al. 1987). Let us compare our computational results with the observation of 4U 1820-30 (Haberl et al. 1987, see Fig. 30).

In Table 8, the burst parameters in the observation and our results are listed. The data of 4U 1820-30 was taken from the observation by *EXOSAT* in 1985 (Haberl et al. 1987) and the theoretical results were for $\dot{M} = 3 \times 10^{-8} M_\odot \, \text{yr}^{-1}$, which are the average values of the two successive bursts shown in Fig. 31, where the luminosity is shown in the solar unit. Also, the persistent luminosity $L_{\text{persistent}}$ in the calculation is assumed to be the observational value ($2 \times 10^{37}$ erg s$^{-1}$).

The calculational results agree well with the burst parameters in the observation ex-
Table 8: Comparison of burst parameters between observation and realistic model calculation.

<table>
<thead>
<tr>
<th></th>
<th>observation (EXOSAT, 1985)</th>
<th>calculation (3 × 10^{-8} M_⊙ yr^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{\text{rise}} ) (s)</td>
<td>3</td>
<td>1.3</td>
</tr>
<tr>
<td>( \tau_{\text{recurrence}} ) (hr)</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>( L_{\text{peak}} ) (erg s^{-1})</td>
<td>2.5 \times 10^{38}</td>
<td>2.6 \times 10^{38}</td>
</tr>
<tr>
<td>( \tau \equiv E_b/L_{\text{peak}} ) (s)</td>
<td>4.5–6.5</td>
<td>6.18</td>
</tr>
<tr>
<td>( \alpha \equiv E_{\text{persistent}}/E_b )</td>
<td>125–155</td>
<td>140.5*</td>
</tr>
</tbody>
</table>

*\( L_{\text{persistent}} \) is assumed to be 2 \times 10^{37} \text{ erg s}^{-1}.

Except for the persistent luminosity. The persistent luminosity estimated for releasing the gravitational energy due to the \( \dot{M} \) in our stellar model is

\[
L_{\text{persistent, cal}} = \frac{GM\dot{M}}{R} = 4.1 \times 10^{38} \text{ erg s}^{-1}. \tag{21}
\]

This is one order of magnitude larger than that of the observation. To obtain the persistent luminosity consistent with both the observation and the other burst parameters, we should consider \( \dot{M} \) in the order of 10^{-9} M_⊙ yr^{-1}. However, the temperature distribution of the neutron star at the initial stage for 10^{-9} M_⊙ yr^{-1} becomes lower than that for 3 \times 10^{-8} M_⊙ yr^{-1}. Therefore, we can not obtain burst parameters, \( \tau_{\text{recurrence}}, \tau \) and \( \alpha \) compared with those of the observation for 10^{-9} M_⊙ yr^{-1}. We need the higher temperature distribution at the initial stage for \( \dot{M} \sim 10^{-9} M_⊙ \text{ yr}^{-1} \). This indicate that, we should include the crust heating (Haensel et al. 1990) which is recently required from the elementary process at high density. Our preliminary calculation indicates that the crust heating does not seem to be insufficient to make initial higher temperature distributions for observation of 4U 1820-30.
Fig. 30.— Background-subtracted 0.9-21.4keV light curve of 4U 1820-30 from 1985 August 19/20. Count rate is in units of counts s$^{-1}$ per half detector array. The seven bursts are spaced at nearly equal intervals of 193 minutes, with a scatter of only ±3 minutes. Adopted from Haberl et al. 1987.

Fig. 31.— Light curve for $\dot{M} = 3 \times 10^{-8} M_\odot \text{ yr}^{-1}$. The interval of two bursts is $1.1 \times 10^4$ s.
7.3. Models of Superbursts

Superbursts observed in 4U 1820-30 and 4U 1636-536, 4U 1735-44 and 4U 1254-690 are considered to be accreted by pure helium and hydrogen/helium as the accretion matter, respectively (see Table 1). The difference of the accretion matter would effect the mechanism of superbursts. Therefore, we first examine rather simplified models of superbursts, which are triggered by a helium flash and a carbon flash. For the carbon flash, we investigate also the more realistic situation with many normal bursts.

7.3.1. Helium Flash Model

The superburst of 4U 1820-30 was observed in September 1999 with the duration of 2.5 hr and the burst energy of $1.4 \times 10^{42}$ erg (Strohmayer & Brown 2001). For $\dot{M} = 3 \times 10^{-10} M_\odot$ yr$^{-1}$, we have obtained the burst energy around $10^{42}$ erg which lasts more than 3 hr (see Fig. 29). Figure 32 shows the temperature distribution against the density of the initial state (blue dotted line) and that of the maximum $L_n$ (red solid line) for $3 \times 10^{-10} M_\odot$ yr$^{-1}$. At the end of the burst, we get $\log \rho = 9$ that corresponds to $\log P = 26.8$.

In Fig. 32, the ignition curve of the $3\alpha$ reaction ($Y = 0.1$) is drawn by a green dashed line. The deflagration temperature defined by equating the dynamical time scale and the nuclear heating time scale ($\tau_{dyn} = \tau_n$) is shown by a brown dotted line. The two time scales are defined by

$$\tau_{dyn} = \frac{H_p}{c_s}, \quad \tau_n = \frac{C_p T}{\varepsilon_n},$$

where $H_p (\equiv -dr/d\ln P)$ denotes the pressure scale height, $c_s$ the sonic velocity and $C_p$ the specific heat under the constant pressure. It is remarkable that the temperature in the layers of $\log \rho = 8.8$ exceeds $\log T = 8.5$ for $L_{n,\text{max}}$ as seen in Fig. 32: the flash may become the deflagration. It needs to perform a dynamical calculation to elucidate how the deflagration develops inside the accretion layers. Although we cannot represent the proper $L_{peak}$, we advocate that the helium flash in low accretion rate can become a possible site of
Fig. 32.— Temperature distributions in $\dot{M} = 3 \times 10^{-10} M_\odot$ yr$^{-1}$ from the initial state (blue dotted line) and to that of the maximum $L_n$ (red solid line). The green dashed line is the ignition curve ($Y = 0.1$) and the brown dotted line indicate that $\tau_{\text{dyn}} = \tau_{\text{3\alpha}}$

the superburst.

7.3.2. Carbon Flash Model

To explain the superburst from 4U 1820-30, unstable burning in pure $^{12}$C layers has been suggested as the plausible site (Strohmayer & Brown 2001). This is unlikely to apply to the superbursts from hydrogen and helium accreting neutron stars because significant accumulation of pure $^{12}$C is not expected. Koike et al. (1999, 2004) and Schatz et al. (1999, 2001) find that only a small amount of $^{12}$C remains after the combined hydrogen/helium burning due to the rp-process, either during steady state burning or during unstable XRBs. On the other hand, Cumming and Bildsten (2001) show that even small mass fraction of $^{12}$C (typically $X(^{12}\text{C}) \approx 0.05 - 0.1$) is enough to trigger the thermonuclear runaway with
the energy comparable to the superbursts. Let us investigate this C-flash with the stellar evolutionary computation.

We have assumed that the accreting matter is $X(^{12}\text{C}) = 0.1$ plus $X(^{56}\text{Ni}) = 0.9$ with $\dot{M} = 3 \times 10^{-8} M_\odot \text{yr}^{-1}$. This $\dot{M}$ is the Eddington accretion rate $\dot{M}_{\text{Edd}}$ for the neutron star of $1.3 M_\odot$ and 8.1 km. Cumming and Bildsten (2001) considered the two $\dot{M}$, 0.3 and 1 $\dot{M}_{\text{Edd}}$. We have included the crust heating (Haensel et al. 1990; see Appendix E) in our stellar code. In addition, the small network is used for simplicity from $^4\text{He}$ to $^{56}\text{Ni}$ (APRN2; Fig. 15) and we include the reaction of $^{12}\text{C} + ^{12}\text{C} \rightarrow ^{24}\text{Mg}$, where the $^{12}\text{C} + ^{12}\text{C}$ reaction rate is taken from NACRE (Angulo et al. 1999).

In Fig. 33, the solid line is the evolitional track and the composition is $X(^{12}\text{C}) \approx 0.09$ at the ignition. This evolitional track shows that the accreted matter supplied settle down into the bottom of the accretion layer. The dotted line means the ignition curve for $X(^{12}\text{C}) = 0.1$ and $X(^{56}\text{Ni}) = 0.9$. The dotted line shows the watershed (Fujimoto et al. 1984) for $X(^{56}\text{Ni}) = 1.0$. It is defined from the equation of the heat flow in the envelope as $\kappa_p \equiv (d \ln \kappa / d \ln P)_T = -1$ ($\kappa$ is the opacity taking into account both the photon diffusion and the electron conduction). The heat flow is naturally directed outward from the bottom of the accreting layers in the radiative region if $\kappa_p > -1$; the upper left region from the line, and inward in the conductive region if $\kappa_p < -1$; the lower right region, respectively (see Appendix F).

As seen in Fig. 33, the C-flash has been ignited around $\log \rho = 9.7$ and $\log T = 8.6$. We have obtained $\varepsilon_{n,max} \approx 10^{23} \text{erg g}^{-1} \text{s}^{-1}$ and $L_{max}$ of only $\sim 10^{36} \text{erg s}^{-1}$. This was caused by the significant heat conduction to the inner region. We note that the ignition point is located below the watershed line. Our result shows that though the ignition occurs at the deep position where much fuel has been stored, the point is too deep to transport the most energy to the outer region beyond the watershed line. The analytic result of Cumming and Bildsten (2001) show that the temperature is much higher and the density is much lower at the ignition for both $\dot{M}$s they adopted. For this scenario to survive, we need the higher
Fig. 33.— Evolutionary track toward the 'carbon flash' for the bottom of the newly accreted layers (red solid line). The green dashed line is the ignition curve with $X(^{12}\text{C}) = 0.1$, and the light-blue dotted line is the watershed line with $X(^{56}\text{Ni}) = 1.0$, respectively. Beyond the watershed to higher density, the heat by nuclear burning transports inward significantly.

Initial temperature distribution, other initial compositions enough to produce heat transfer, and the stronger crust heating. It should be noted that the equation of state we have used is very soft; a hard one could give the diverse result due to the different ignition condition, because the neutron star radius is larger than that obtained by the soft EOS.
7.3.3. New Model accompanied with Normal Bursts of combined Hydrogen and Helium Burnings

(i) Gross features toward the carbon flash

Let us make an initial model to simulate a superburst with the accretion rate of $5 \times 10^{-9} M_\odot \, \text{yr}^{-1}$. This accretion rate is considered to be the mean value in the sources where superbursts are observed. With the nuclear burning suppressed, we construct an initial temperature distribution by continuous accretion with this specified $\dot{M}$. Compositions in the accretion matter are assumed to be H (73.0%), $^4$He (25.0%), $^{14}$O (0.7%) and $^{15}$O (1.3%). The bottom of the accretion layer is set to be pure $^{56}$Ni, which is equivalent to $^{56}$Fe in the present purpose. The steady state concerning the accretion rate is assumed, that is, $\dot{M} = \dot{M}_{56\text{Ni}}$; this means that accreted matter increases the mass of the layer of pure $^{56}$Ni. At the end, we obtain the isothermal temperature distribution of $\log T \sim 8.42$.

We use the approximation network developed for hydrogen/helium burning (APRN1; Fig. 13). We repeatedly calculated normal bursts (combined hydrogen/helium burning) with use of the evolutionary code which includes the approximation reaction network. The totally calculated evolutionary time is $1.81 \times 10^9$ s and the total number of normal bursts amounts to 2786. Our time sequence of the bursts is illustrated in Fig. 34. The time interval $\Delta t$ and the number of bursts are given in Table 9 for the individual period specified in terms of $\dot{M}$. Figures 35 – 37 shows the light curve during the intervals (3 – 11), (11 – 15) and $(17.6 – 18.1) \times 10^8$ s. Critical bursts are marked by $\#1$–$7$, whose profiles are presented later.

Important epochs for a superburst are specified as follows: The epoch $*\alpha$ is 930 s after the burst at $2 \times 10^8$ s during “period 1”, the epoch $*\beta$ is 854 s after the burst at $1.5 \times 10^9$ s during “period 2”. the epoch $*\gamma$ is the end of the last burst $\#7$ and $*\delta$ is just before the $^{12}\text{C} + ^{12}\text{C}$ ignition which is 5500 s after the epoch $*\gamma$.

Figures 38 and 39 show temperature profiles against density and pressure, respectively, at the epochs $*\alpha - *\gamma$. The ignition curve and deflagration line of the $^{12}\text{C} + ^{12}\text{C}$ reaction
Fig. 34.— Time sequences of the models accompanied with normal bursts. When we start the accretion on the neutron star, time is set to be zero.

![Accretion rate and time sequence diagram](image)

Table 9: Time sequences of bursts: $\dot{M}$, time interval ($\Delta t$) and number of bursts for periods 1-3.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\dot{M}$ ($M_\odot$ yr$^{-1}$)</th>
<th>Time ($10^8$ s)</th>
<th>$\Delta t$ ($10^8$ s)</th>
<th>Number of bursts</th>
<th>Specific stages of bursts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5 \times 10^{-9}$</td>
<td>0 – 12</td>
<td>12</td>
<td>1237</td>
<td>$\alpha$, $\beta_1-6$</td>
</tr>
<tr>
<td>2</td>
<td>$1 \times 10^{-9}$</td>
<td>12 – 16.8</td>
<td>4.8</td>
<td>1416</td>
<td>$\beta$</td>
</tr>
<tr>
<td>3</td>
<td>$5 \times 10^{-9}$</td>
<td>16.8 – 18.1</td>
<td>1.3</td>
<td>133</td>
<td>$\beta_7$, $\gamma$, $\delta$</td>
</tr>
</tbody>
</table>

are also shown in Fig. 38. The energy generation rate is shown in Fig. 40. It should be noted that the curve of $\varepsilon_n$ for $\delta$ falls down steeply around $\log P = 26 - 26.5$ because of the significant decrease in abundances due to the sudden development of the convection as illustrated in Fig. 44.

For “period 1”, the accretion rate is set to be $5 \times 10^{-9}M_\odot$ yr$^{-1}$ from the beginning of the accretion to $1.2 \times 10^9$ s. The number of bursts occurred in this interval is 1237. The temperature distribution inside the neutron star’s core ($\log \rho \geq 8$ and $\log P \geq 25$) remains isothermal of $\log T \sim 8.44$ (see the temperature distribution at $2 \times 10^8$ s in Figs. 38 and 39). We can recognize that unstable combined hydrogen/helium burning has been generated for $\log \rho = 6 - 6.5$ and $\log P = 22.5 - 23.5$. Figure 41 shows the composition distribution at $2 \times 10^8$ s ($\alpha$). The range of the pressure is equal to in Fig. 39. The correspondence between
lines and compositions in Fig. 41 is shown in the following Table 10. Around \( \log P = 22 - 23 \), the rp-process produces both \( ^{68}\text{Ge} \) and \( ^{64}\text{Zn} \). Though there does not exist remained fuel of hydrogen, remained helium produces \( ^{12}\text{C} \) for \( \log P > 24 \) (see the violet solid line in Fig. 41) due to the steady burning. The increase in \( ^{56}\text{Ni} \) is ascribed to the numerical diffusion of the initial distribution and convective mixing at the beginning of the accretion for \( \log P > 26 \): thus, we can consider that the mass fraction of \( ^{56}\text{Ni} \) in \( 26 \leq \log P \leq 27.5 \) should be added to \( ^{68}\text{Ge} \). For “period 1”, the carbon burning is stable (\( \log P \sim 28 \), \( \log T \sim 8.4 \) and \( X(^{12}\text{C}) \leq 0.01 \)) in the sense that the increase in \( T \) does not reach the ignition curve. In the bottom of the accretion layer (\( \log P \sim 28 \)), carbon sometimes burns intensively and the temperature of the layer has increased to \( \log T \geq 8.6 \). The amount of carbon is too small \( (X(^{12}\text{C}) < 0.01) \) to trigger the nuclear flash. The nuclear energy generation \((\ast\alpha)\) is supplied by helium burning for \( \log P = 23 - 25 \), while it is supplied by reactions such as \( ^{12}\text{C}(\alpha, \gamma)^{16}\text{O} \) and \( ^{12}\text{C} + ^{12}\text{C} \) for \( \log P = 25 - 27.5 \). The crust heating help \( \varepsilon_n \) to increase again for \( \log P > 27.5 \).

Table 10: Correspondence between lines and compositions.

<table>
<thead>
<tr>
<th>color</th>
<th>solid line</th>
<th>dotted line</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>green</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>( ^{4}\text{He} )</td>
</tr>
<tr>
<td></td>
<td>( ^{30}\text{S} )</td>
<td>( ^{56}\text{Ni} )</td>
</tr>
</tbody>
</table>

After \( t = 1.2 \times 10^9 \) s the accretion rate is artificially changed to \( 1 \times 10^{-9} M_\odot \text{ yr}^{-1} \). This results in the increase in \( ^{12}\text{C} \) abundance. We keep this accretion rate in the interval of \( 4.8 \times 10^8 \) s, and the 1416 bursts occur during this interval (“period 2”). The temperature distributions for this accretion rate is shown by the line of \( 1.5 \times 10^9 \) s in Figs. 38 and 39 (denoted by \( \ast\beta \)). Figure 42 shows the composition distribution at \( 1.5 \times 10^9 \) s (\( \ast\beta \)). For “period 2”, hydrogen is consumed completely in the region of \( \log P > 23 \) and the remained helium has increased \( ^{12}\text{C} \) appreciably. The rp-process first produces \( ^{68}\text{Ge} \) and \( ^{64}\text{Zn} \), and afterwards also \( ^{60}\text{Ni} \) and \( ^{56}\text{Ni} \). The appreciable decrease in \( \varepsilon_n \) corresponds to the dip of the
composition distribution around $\log P \sim 27.4$ in Fig. 42.

We changed again the accretion rate to $\dot{M} = 5 \times 10^{-9} M_\odot \, \text{yr}^{-1}$ and kept it for $1.3 \times 10^8$ s (“period 3”). The number of bursts is 133 in this interval. Since the temperature distribution becomes lower compared with “period 1”, we increase the crust heating by a factor of 10 to save the computational time: it can be considered that if we continue the burst calculations, the temperature distribution should be recovered to that in “period 1”. Finally, the $^{12}\text{C} + ^{12}\text{C}$ reaction is successfully ignited in $\log P = 28$ at the epoch $(1.81 \times 10^9 \, \text{s})$. Figure 43 shows the composition distribution at this stage. As in the case of “period 1”, hydrogen is consumed in the bottom region of the hydrogen/helium burning and the remained helium forms $^{12}\text{C}$ continuously below the region. After $5500 \, \text{s}$ ($\ast \delta$) from the end ($\ast \gamma$) of the last burst ($\ast \gamma$), we obtain the carbon flash. The composition distributes uniformly for $\log P = 26 - 28$ due to the convection as shown in Fig. 44. We note that the $^{12}\text{C} + ^{12}\text{C}$ flash does not develop to the peak of the total nuclear burning rate.
Fig. 35.— Light curve at $(3 - 7) \times 10^8$ s (top) and those from $(7 - 11) \times 10^8$ s (bottom). *α corresponds to the same epoch as shown by the mark in Fig. 38. The interval between $(8.25 - 9.75) \times 10^8$ s is named the “log interval 1”. Marks of #1-5 and #6 are successive numbers and a sign of each burst before and after “long interval 1”, respectively.
Fig. 36.— Light curve at $(11 - 15) \times 10^8$ s. $*\beta$ indicates the same epoch as the mark in Fig. 38.

Fig. 37.— Light curve at $(17.6 - 18.1) \times 10^9$ s. $\#7$ indicate the last burst before the ignition of the carbon flash ($*\gamma$ and $*\delta$).
Fig. 38.— Temperature distribution against density at $2\times10^8$ s (blue solid line; *α), 1.5×10⁹ s (green dotted line; *β), 1.81×10⁹ s (orange dashed line; *γ) and the 5500 s from the *γ (red dot-dashed line; *δ). At the stage *δ, the $^{12}\text{C}+^{12}\text{C}$ reaction is ignited. Two lines on the right are ignition curves of $^{12}\text{C}+^{12}\text{C}$ reaction (green dashed line, $\varepsilon_{\text{rad}} = \varepsilon_{^{12}\text{C}+^{12}\text{C}}$), and deflagration line (light blue dashed line, $\tau_{\text{dyn}} = \tau_{^{12}\text{C}+^{12}\text{C}}$), with $X(^{12}\text{C})=0.1$, $X(^{56}\text{Ni})=0.9$. The two convexities near log $\rho = 6 - 7$ are due to the effect of the unstable hydrogen/helium burning.

Fig. 39.— Same as Fig. 38 but for the pressure.
Fig. 40.— Nuclear energy generation rate vs. pressure for four cases as shown in Figs. 38 and 39. Around log $P = 27 - 28$, the energy generation rates of $\ast\alpha$, $\ast\beta$, and $\ast\gamma$ before the $^{12}\text{C} + ^{12}\text{C}$ ignition $\ast\delta$ become small because there remains small amount of fuel as the result of both the rp-process and steady burning of the $^{12}\text{C} + ^{12}\text{C}$ reaction. Hot-CNO cycle has produced the energy of $10^{14}$ erg s$^{-1}$ for log $P \leq 22$. 
Fig. 41.— Composition distribution at $2 \times 10^8$ s (*$\alpha$). For $\log P = 22 - 23$, the rp-process occurred and $^{68}$Ge and $^{64}$Zn were produced. For the relation between lines and composition, see Table 10.

Fig. 42.— Composition distribution at $1.5 \times 10^9$ s (*$\beta$). The rp-process formed $^{68}$Ge, $^{64}$Zn and $^{60}$Ni.
Fig. 43.— Composition distribution at $1.81 \times 10^9$ s ($\gamma$). For $\log P = 21.6 - 23.3$, the amount of ${}^{68}\text{Se}$ and ${}^{64}\text{Ge}$ became large temporarily due to the hydrogen/helium mixed burning.

Fig. 44.— Composition distribution just after the $^{12}\text{C} + ^{12}\text{C}$ ignition ($\delta$). There exists the region of the uniform composition distribution for $\log P = 26 - 28$. 
(ii) Detailed features of key bursts that affect the carbon flash

We discuss the profiles of the bursts for each period in detail. In Fig. 35 (bottom), there is a significant interval at $8.25 - 9.75 \times 10^8$ s. We call it the “long interval 1”. We are interested in the five bursts ($\#1$-5) before the “long interval 1” and the first burst ($\#6$) after the interval.

Five bursts $\#1$-5 extracted from Fig. 35 are shown in Fig. 45. “Long interval 1” begins at $8.252 \times 10^8$ s. The light curve of the burst $\#1$ is shown in Fig. 46. In Fig. 46, we set the time to be zero when the burst $\#1$ start. The roman numerals specify the epochs; I: just before the convection, II: peak of the total nuclear burning rate $L_{n,max}$, III: 1/10 of the maximum luminosity $L_{max}$. The total nuclear burning rate $L_n$ of the burst $\#1$ is shown in Fig. 47. Composition distributions in each epoch of I–III are shown in Figs. 48–50, respectively. In the same manner, the profiles of the burst $\#2$-6 are shown in Figs. 52–83, where the burst $\#6$ is the next burst after the “long interval 1”. The epoch 'IV' for the bursts $\#3$ and $\#5$ are 2240 and 220 s, respectively, after the onset of each burst. The profiles of the burst at $\ast \beta$ with $\dot{M} = 1 \times 10^{-9} M_\odot$ yr$^{-1}$ are shown in Figs. 84–89. The last burst $\#7$ before the $^{12}$C$+^{12}$C ignition (see Fig. 37) are shown in Figs. 90–95. Figures 96 and 97 show the light curves, and Fig. 98 shows total nuclear burning rate of the burst $\#7$ and the beginning of the $^{12}$C$+^{12}$C ignition (see also Table 11 and Fig. 34).

The shapes of the light curves and the products for the bursts $\#1$-4 and $\#6$ are almost same. The burst $\#5$ is the longest burst compared with other bursts of $\#1$-4 and $\#6$ (see Fig. 71). At 'IV’ of the burst $\#5$, the hydrogen burns in $\log P \leq 22.5$, and $^{60}$Zn is produced (see Fig. 76). The convection occurs for $\log P = 22.2 - 23.2$ and hydrogen is completely consumed. The heat from this burning region of the convection is the reason for the long burst as seen in Fig. 71, and the consumption of fuels leads to the long interval after $\#5$ [see Figs. 35 (bottom) and 45]. This feature can be seen in other sequences of bursts. At the epoch 'IV’ of the burst $\#3$, the convection also occurs between $\log P = 22.5 - 23$ and hydrogen is consumed completely (see Fig. 63). Therefore, as seen in Fig. 45 the interval
Fig. 45.— Light curve at $8.15 - 8.27 \times 10^8$ s. Each number from $\sharp 1 - 5$ corresponds to each burst. “Long interval 1” begins after the elapsed time $8.252 \times 10^8$ s ($\sharp 5$).

to the next burst $\sharp 4$ becomes long compared to other intervals except for the “long interval 1”, because the composition in the accumulated hydrogen consumed (compare Fig. 63 with Fig. 56). The same situation also occurs for the rather long interval $1.14 - 1.19 \times 10^9$ s in Fig. 36.

The luminosity (Fig. 84) in the burst $*\beta$ of $\dot{M} = 1 \times 10^{-9} M_\odot$ yr$^{-1}$ is a little small compared with the bursts $\sharp 1$-$5$ and $\sharp 6$ of $\dot{M} = 5 \times 10^{-9} M_\odot$ yr$^{-1}$. Though the products of $^{68}$Ge, $^{68}$Se, $^{64}$Zn and $^{22}$Mg are the same in the other bursts, the amount of heavy elements like $^{60}$Ni and $^{64}$Zn are produced more than those of the other bursts. As seen in Fig. 89, the temperature distribution of the inner accretion layer ($\log P > 24$) is lower than that of the other bursts with $\dot{M} = 5 \times 10^{-9} M_\odot$ yr$^{-1}$ (see also Figs. 38 and 39, where $T$ of $*\beta$ for $\log P > 24$ is low due to the conduction). We can consider that due to the increased
conduction the luminosity in the bursts decreased because the heat from the combined hydrogen/helium burning flowed into the core. The last burst ♯7 before the carbon ignition is weaker in strength than that of the burst *β, because the hydrogen/helium burning is ignited at the deeper region (log $P = 23.3$) than that to the other bursts (log $P = 22.9$).

In Figs. 96–98, the burst ♯7 and the onset of the carbon flash are shown. Just after the burst, *γ, the convection occurs at the narrow region of log $P = 27.9 – 28$ (Fig. 43). At $\sim 5300$ s in Fig. 98, the convection begins to spread to lower pressure region, and it extends from log $P = 28$ to 26 as shown in Fig. 44. We note that $L_n$ at *δ is not attained to the $L_{n,max}$. However, since the temperature in log $P \geq 27.8$ increased beyond the deflagration temperature defined by $\varepsilon_n = \varepsilon_{dyn}$ as seen from Fig. 39, we can infer the flash should develop to a dynamical phenomena of deflagration. Therefore, we stop the calculation of the light curve of a superburst due to the numerical difficulty; nevertheless we suggest the possibility that this flash becomes a superburst.

Table 11: Correspondent relations between the burst number (♯) & each epoch (*), $L$, $L_n$, and numbers of figures 46 – 98.

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<thead>
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<th>Burst &amp; Epoch</th>
<th>$L/L_\odot$</th>
<th>$L_n/L_\odot$</th>
<th>I (before convection)</th>
<th>II ($L_{n,max}$)</th>
<th>III (1/10 of $L_{max}$)</th>
<th>IV</th>
<th>log $T$ (K)</th>
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<tr>
<td>7</td>
<td>90</td>
<td>91</td>
<td>92</td>
<td>93</td>
<td>94</td>
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<tr>
<td>$\gamma$, $\delta$</td>
<td>96, 97</td>
<td>98</td>
<td></td>
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</tr>
</tbody>
</table>

*♯ 2240 s after the onset of the burst ♯3
**$^b$ 220 s after the onset of the burst ♯5
Fig. 46.— Light curve of the burst ♯1. Roman numerals are corresponded with the time of Fig. 48−51. I: just before the start of convection, II: $L_{n,\text{max}}$, III: 1/10 of $L_{\text{max}}$.

Fig. 47.— Total nuclear burning rate of the burst ♯1. Roman numerals are the same as in the Fig. 46.
Fig. 48.— Composition distribution of the burst $\tau 1$ at the time just before the convection occurs (I).

Fig. 49.— Same as Fig. 48 but for the time of $L_{n,\text{max}}$ (II).
Fig. 50.— Same as Fig. 48 but for the time of 1/10 of $L_{\text{max}}$ (III).

Fig. 51.— Temperature distribution of the burst ‡1 for I–III.
Fig. 52.— Same as Fig. 46 but for the burst \#2.

Fig. 53.— Same as Fig. 47 but for the burst \#2.
Fig. 54.— Composition distribution of the burst $\sharp 2$ at the time just before the convection occurs (I).

Fig. 55.— Same as Fig. 54 but for the time of $L_{n,max}$ (II).
Fig. 56.— Same as Fig. 54 but for the time of 1/10 of $L_{\text{max}}$ (III).

Fig. 57.— Temperature distribution of the burst $\#2$ for I–III.
Fig. 58.— Same as Fig. 46 but for the burst \#3. IV is the time of 2240 s after the onset of the burst.

Fig. 59.— Same as Fig. 47 but for the burst \#3.
Fig. 60.— Composition distribution of the burst \( \xi_3 \) at the time just before the convection occurs (I).

Fig. 61.— Same as Fig. 60 but for the time of \( L_{n,\text{max}} \) (II).
Fig. 62.— Same as Fig. 60 but for the time of 1/10 of $L_{\text{max}}$ (III).

Fig. 63.— Same as Fig. 60 but for the time of 2240 s after the onset of the burst $\sharp 3$ (IV).
Fig. 64.— Temperature distribution of the burst §3 for I–IV.
Fig. 65.— Same as Fig. 46 but for the burst ♯4.

Fig. 66.— Same as Fig. 47 but for the burst ♯4.
Fig. 67.— Composition distribution of the burst $\sharp 4$ at the time just before the convection occurs (I).

Fig. 68.— Same as Fig. 67 but for the time of $L_{n,\text{max}}$ (II).
Fig. 69.— Same as Fig. 67 but for the time of 1/10 of $L_{max}$ (III).

Fig. 70.— Temperature distribution of the burst #4 for I–III.
Fig. 71.— Same as Fig. 46 but for the burst ¶5. IV is the time of 220 s after the onset of the burst. This is a long burst compared with other burst ¶1-4. For comparison, the burst ¶1 is also shown.

Fig. 72.— Same as Fig. 47 but for the burst ¶5.
Fig. 73.— Composition distribution of the burst $\sharp 5$ at the time just before the convection occurs (I).

Fig. 74.— Same as Fig. 73 but for the time of $L_{n,\text{max}}$ (II).
Fig. 75.— Same as Fig. 73 but for the time of 1/10 of $L_{\text{max}}$ (III).

Fig. 76.— Same as Fig. 73 but for the time of 220 s after the onset of the burst 5 (IV).
Fig. 77.— Temperature distribution of the burst §5 for I–IV.
Fig. 78.— Same as Fig. 46 but for the burst \#6.

Fig. 79.— Same as Fig. 47 but for the burst \#6.
Fig. 80.— Composition distribution of the burst $\xi6$ at the time just before the convection occurs (I).

Fig. 81.— Same as Fig. 80 but for the time of $L_{n,\text{max}}$ (II).
Fig. 82.— Same as Fig. 80 but for the time of 1/10 of $L_{\text{max}}$ (III).

Fig. 83.— Temperature distribution of the burst 6 for I–III.
Fig. 84.— Same as Fig. 46 but for the burst $\beta$.

Fig. 85.— Same as Fig. 47 but for the burst $\beta$. 
Fig. 86.— Composition distribution of the burst *β at the time just before the convection occurs (I).

Fig. 87.— Same as Fig. 86 but for the time of $L_{n,max}$ (II).
Fig. 88.— Same as Fig. 86 but for the time of $1/10$ of $L_{\text{max}}$ (III).

Fig. 89.— Temperature distribution of the burst $*\beta$ for I–III.
Fig. 90.— Same as Fig. 46 but for the burst 

Fig. 91.— Same as Fig. 47 but for the burst 

The III is the time of 1/10 of \( L_{\text{max}} \) and 154 s after the onset of the burst 

Fig. 91.— Same as Fig. 47 but for the burst 


Fig. 92.— Composition distribution of the burst $7$ at the time just before the convection occurs (I).

Fig. 93.— Same as Fig. 92 but for the time of $L_{n,max}$ (II).
Fig. 94.— Same as Fig. 92 but for the time of 1/10 of $L_{\text{max}}$ (III).

Fig. 95.— Temperature distribution of the burst $\sharp 7$ for I–III.
Fig. 96.— Same as Fig. 46 but for the burst ♯7 and the onset of the carbon flash. ♯γ and ♯δ corresponds to the same epoch as shown by the mark in Fig. 38.

Fig. 97.— Same as Fig. 96 but for the logarithmic scale.
Fig. 98.— Same as Fig. 47 but for the burst #7 and the onset of the carbon flash.
8. Summary

We performed post process calculations for successive two bursts with use of a large nuclear reaction network. Since we used the density and temperature obtained from the evolutionary calculations, it was possible to take into account the effects of the multi-zone and convection. We confirmed that the products of first burst and second burst were different for $\dot{M} = 3 \times 10^{-9}$ and $3 \times 10^{-10} M_\odot \text{ yr}^{-1}$. Main products of the second burst are lighter in mass than those of the first burst for each case.

We have presented the three cases for the models of superbursts: helium flash, single carbon flash and carbon flash accompanied with many normal bursts. We studied the carbon flash for a superburst based on the scenario by Cumming & Bildsten (2001). For the case of helium flash, the burst by the flash had a long duration time although the accretion rate is different from the observation. We suggest the possibility that helium flash can originate the superburst. For the case of a single carbon flash, the burst did not show the long duration time. However, as the temperature of the heated layer became very high, the burning could become dynamical. Because our calculations were performed under the assumption of the hydrostatic equilibrium, it does not clear for the burning to become dynamical. We inferred that the burning might become a long burst due to the dynamical burning.

For the case of carbon flash accompanied with many normal bursts, we carried out the successive 2786 normal bursts up to the time of $1.81 \times 10^9$ s using the observed accretion rates. We showed the profiles of the several normal bursts and the onset of the carbon flash after the normal bursts. We knew that the recurrence time of bursts became longer than those of the earlier bursts since the products existed below the layer where hydrogen/helium burning had occurred. We presented that the recurrence time of bursts became longer than other bursts when the little burning occurred after the main burning for the bottom of the layer where hydrogen/helium burning had occurred. The carbon flash also could become dynamical such as the case of the single carbon flash. Since normal bursts have been observed
before the superbursts, our scenario is consistent with the observations. We conclude that a carbon flash should trigger a superburst.
9. Appendix

A. Rapid Proton Capture Process

Nuclear burning in the region of high density in XRBs and novae proceeds by way of proton captures, $\beta^+$ decays, and alpha-induced reactions on unstable proton rich nuclei. This nuclear process has been referred to as the rapid proton capture process (rp-process; Wallace and Woosley 1981).

For a hot dense mixture of hydrogen, helium and a trace of CNO isotopes, with the temperature in the range $0.2 \leq T_9 \leq 0.5 \ (T_9 = T/10^9 \text{ K})$, these elements can quickly adjust themselves to a steady state with most material of metals consisted of $^{14}\text{O}$ and $^{15}\text{O}$. Then, nuclear energy generation arises from the Hot-CNO cycle: $^{14}\text{O}(e^+, \nu)^{14}\text{N}(p,\gamma)^{15}\text{O}(e^+, \nu)^{15}\text{N}(p, \alpha)^{12}\text{C}(p,\gamma)^{13}\text{N}(p,\gamma)^{14}\text{O}$, at a rate fixed by the mean decay lifetime of $^{14}\text{O} \ (102 \text{ s})$ and $^{15}\text{O} \ (176 \text{ s})$,

$$\varepsilon_{\text{Hot-CNO}} = 5.86 \times 10^{15} Z' \ \text{erg g}^{-1} \text{ s}^{-1}, \quad (A1)$$

where $Z'$ denotes the sum of the initial mass fraction of CNO isotopes, and 2.03 MeV per cycle has been subtracted to account for neutrinos emitted by the $e^+$ decays of $^{14}\text{O}$ and $^{15}\text{O}$. For $T \geq 5 \times 10^8 \text{ K}$ leakage out of the Hot-CNO cycle is initiated by $^{15}\text{O}(\alpha, \gamma)^{19}\text{Ne}$ and $^{14}\text{O}(\alpha, p)^{17}\text{F}$ reactions. Once $^{19}\text{Ne}$ has captured a proton, it is impossible for reaction chains to return to the Hot-CNO loop.

While the path followed by the ensuing chain of proton capture is sensitive to the composition and temperature, its qualitative nature is very similar to the classical “r-process”. The hot hydrogen environment converts seed nuclei into isotopes around the region of proton-unbound nuclei for which further proton capture is hindered by Coulomb barrier and photodisintegration. For each neutron number, a maximum mass number is reached where the flow is stopped by a weak interaction, $e^+$ decay, before the build up of still heavier elements. This process is called the “rp-process”.

Recently, the rp-process is found to terminate in the mass region $\sim 100$ by the block of the SnSbTe cycle (Schatz et al. 2001). The isotopes of $^{106}-^{109}\text{Te}$ are experimentally known.
to be $\alpha$-emitters, because their $\alpha$ emission $Q$-values are $\simeq 3 - 4$ MeV. In particular, the $(\gamma, \alpha)$ reaction of $^{107}$Te would be the key reaction to determine the endpoint of the rp-process.

**B. Evolutionary Equations for the Accreting Neutron Star**

The general relativistic evolutionary equations of spherical stars in hydrostatic equilibrium, as formulated by Thorne (1977), are written as

\begin{align}
\frac{\partial M_{tr}}{\partial r} &= 4\pi r^2 \rho_t \\
\frac{\partial P}{\partial r} &= -\frac{G M_{tr} \rho_t}{r^2} \left(1 + \frac{P}{\rho_t c^2}\right) \left(1 + \frac{4\pi r^3 P}{M_{tr} c^2}\right) \mathcal{V}^2 \\
\frac{\partial(L_r e^{2\phi/c^2})}{\partial M_r} &= e^{2\phi/c^2} \left(\varepsilon_n - \varepsilon_\nu - e^{-\phi/c^2} T \frac{\partial s}{\partial t_\infty}\right) \\
\frac{\partial \ln T}{\partial \ln P} &= \min(\nabla_{rad}, \nabla_{ad}) \\
e^{-\phi/c^2} \frac{\partial Y_i}{\partial t_\infty} &= \alpha_i \\
\frac{\partial M_{tr}}{\partial M_r} &= \frac{\rho_t}{\rho} \mathcal{V}^{-1} \\
\frac{\partial \phi}{\partial M_{tr}} &= \frac{G (M_{tr} + 4\pi r^3 P/c^2) }{4\pi r^4 \rho_t} \mathcal{V}^2,
\end{align}

where \( \mathcal{V} \equiv \left(1 - \frac{2GM_{tr}}{c^2 r}\right)^{-1/2} \).

The basic quantities are defined as follows: \( t \): Schwarzschild time coordinate (proper time at a distant observer), \( M_r \): proper mass inside the radius \( r \), \( M_{tr} \): total mass inside the radius \( r \), \( \rho \): rest mass density, \( \phi \): gravitational potential, \( \rho_t \): total nongravitational mass-energy density in mass units, \( s \): specific entropy, \( \alpha_i \): nuclear reaction rate for the \( i \)-th particle which is the same as the right hand side of the rate equation (19), \( \nabla_{rad} (\nabla_{ad}) \): the radiative (adiabatic) temperature gradient.

Eq. (B2) is the TOV (Tolman, Oppenheimer, Volkoff) equation. Under the assumption of hydrostatic equilibrium, we can simulate the XRB with the coupled equations, (B1) – (B7), starting from an appropriate initial model. We show the flow chart of our evolution code in Fig. 99.
Fig. 99.— Flow chart of the code. The subscripts 0 and 1 mean the value of the present stage and the next one. $\mu_I$ and $\mu_e$ are the molecular weight per ion and electron, respectively.
C. Basic Idea of the Shell Flash Model

C.1. One-zone Model Equations

An envelope of the neutron star can be considered to be geometrically thin during bursts, because the value of absolute the gravitational energy is much larger than the released nuclear energy (Fujimoto et al. 1981). Therefore, the plane-parallel configuration is a good approximation.

When the effect of the general relativity is taken into account (Hanawa & Fujimoto 1982), we introduce the two independent variables, the proper time \( t \) for the observer at the surface of the star and the column density \( \sigma \) of the rest mass lying above a shell. They are related to the Schwarzschild time coordinate \( t_\infty \) and the proper mass \( \Delta M \) of the overlying layers by

\[
\frac{dt}{d\sigma} = V^{-1} dt_\infty, \tag{C1}
\]

\[
\sigma = -\frac{\Delta M}{4\pi R^2}, \tag{C2}
\]

where \( V \) is the volume correction factor (B8). With (C1), the energy equation (B3) is

\[
T \frac{\partial s}{\partial t} = \varepsilon_n - \varepsilon_\nu + \frac{\partial F_r}{\partial \sigma}, \tag{C3}
\]

where \( F_r = L_r/4\pi R^2 \) is the heat flux. For radiative equilibrium, \( F_r \) is given by

\[
F_r = \frac{c}{\kappa} \frac{\partial}{\partial \sigma} \left( \frac{aT^4}{3} \right). \tag{C4}
\]

The gradient of the flux is approximated

\[
\frac{\partial F_r}{\partial \sigma} = -\varepsilon_{\text{rad}} = -\frac{4ac}{3\kappa} \frac{T^4}{\sigma^2}. \tag{C5}
\]

Then, the energy equation (C3) is reduced to

\[
\epsilon_p \frac{dT}{dt} = \varepsilon_n - \varepsilon_{\text{rad}}, \tag{C6}
\]
where the neutrino loss is neglected though the $\beta$ associated neutrino loss is not always neglected. The neutrino loss rate for the $\beta$-decays would be less than 20 percent of $\varepsilon_n$ around the peak of the burst.

We derive the equation of hydrostatic equilibrium under the plane-parallel approximation. For a thin envelope on the surface of the star, the factor of $e^{-\phi/c^2}$ becomes $(1 - 2GM/c^2R)^{-1/2}$ and constant due to $M_t \simeq M$. Furthermore, the term of $P/\rho c^2$ and $4\pi r^3P/Mc^2$ in Eq. (B1) can be negligible. From Eq. (B1) and Eq. (B6), the hydrostatic equation reduces to

$$\frac{\partial P}{\partial M_r} = -\frac{GM}{4\pi R^3} V. \quad (C7)$$

Introducing the variable $d\sigma = -dM_r/4\pi R^2$ with the boundary condition of $P = 0$ at $\sigma = 0$, the integration of (C7) gives

$$P = \frac{\Delta MV^{-1}}{4\pi R^2} \frac{GM}{R^2 V^{-2}} = g_\sigma \sigma \quad (C8)$$

The pressure of the burning layer is determined by the weight of the overlying layers. This indicates that the flash proceeds at the constant pressure.

### C.2. Nuclear Burning under the Constant Pressure

The configuration of the shell in the envelope depends on the ratio $r$ to $H_p$ (Sugimoto & Fujimoto 1978),

$$V \equiv \frac{r}{H_p} = -\frac{d\ln P}{d\ln r} = \frac{GM\rho}{Pr} \quad (C9)$$

The pressure of the shell is written by $V$ and the polytrope index $N$ as

$$P = g_\sigma \sigma f(V,N), \quad (C10)$$

$$\frac{N}{N+1} = -H_p \frac{d\ln \rho}{dln r}, \quad (C11)$$

where $f(V,N)$ is a *flatness parameter* that represents the geometric configuration of the shell. When the shell has the flat configuration, $f$ becomes unity. The *flatness parameter*
can be expressed by the infinite series:

\[
[f(V, N)]^{-1} = \sum_{k=0}^{\infty} b_k, \quad b_0 = 1, \quad b_k = b_{k-1} \frac{k + 3}{N + k + 1} \left( \frac{N + 1}{V} \right).
\] (C12)

Because of the large gravitational potential, \( V \) is very large in the envelope of the neutron star: \( H_p = \sigma/\rho = 1 - 10^2 \), \( V = 10^4 - 10^6 \) for \( r = 10 \text{km} \). Even at the peak of bursts, it remains as large as \( V \sim 10^3 \). This implies that such shells are in flat configuration: \( P \) is determined only by the weight of the overlying layers without regard to the thermal state of the envelope (Hara et al. 1976). Though we have considered for the non-relativistic case, the configuration of the shell flash would not be changed even if we take account of the general relativistic effect.

### D. Criterion of the Convection

![Diagram of convection](image)

Fig. 100.— Stability for convection with a blob adiabatically moved from \( r_1 \) to \( r_2 \).

The notations of this appendix are followed from Kippenhahn and Weigert. Consider a blob to move from some point within a star, as shown in Fig. 100. The criterion for convective stability is written as

\[
\nabla_{\text{rad}} < \nabla_{\text{ad}} + \frac{\varphi}{\delta} \nabla_{\mu},
\] (D1)

\[
\nabla_{\text{rad}} \equiv \left( \frac{d\ln T}{d\ln P} \right)_s, \quad \nabla_{\text{ad}} \equiv \left( \frac{d\ln T}{d\ln P} \right)_{ad}, \quad \nabla_{\mu} \equiv \left( \frac{d\ln \mu}{d\ln P} \right)_s.
\] (D2)
with
\[
\delta = - \left( \frac{d \ln \rho}{d \ln T} \right)_{P, \mu}, \quad \varphi = - \left( \frac{d \ln \rho}{d \ln \mu} \right)_{P, T} \tag{D3}
\]
Here \( \mu \) is the mean molecular weight and the subscripts indicate that the derivatives are to be taken in the surrounding pressure. The temperature gradient \( \nabla_{\text{rad}} \) indicates that the energy is transported by radiation (or conduction) only. This formula is known as the Ledoux criterion for dynamical stability. In a region with homogeneous distribution of chemical compositions, \( \nabla \mu = 0 \). Then we obtain the Schwarzschild criterion for dynamical stability:
\[
\nabla_{\text{rad}} < \nabla_{\text{ad}}. \tag{D4}
\]
If in the criteria (D1) or (D4) the left-hand side is larger than the right, the layer is dynamically unstable, which causes a convective motion. If they are equal, it is called marginally stable or ‘neutral’. The two criteria exhibit a difference if there exists a region where the chemical composition varies; heavier nuclides are usually produced below the lighter ones due to nuclear burning, such that the \( \mu \) increases inwards and \( \nabla \mu > 0 \). Since the last term in inequality (D1) becomes positive according to \( \varphi > 0 \) and \( \delta > 0 \), the criterion indicates to increase the stability. This inhibits a convection, because the blob carried upwards into lighter surroundings will tend to draw it back to its original place by gravity. We see that such regions occur in the interior of evolving stars. It can be considered that effects of \( \nabla \mu \) may not be important if the convection occurs efficiently.

E. Crustal Heating

Temperature distributions in the core and crust of an accreting neutron star are a subject of current interest, from the point of magnetic field evolution and thermonuclear burning (Brown & Bildsten 1998). For neutron stars that steadily accrete with \( \dot{M} \sim 10^{-11} - 10^{-9} M_\odot \) yr\(^{-1} \), the recurrent heating from thermally unstable hydrogen/helium burning balances with the cooling from neutrino emission and radiative diffusion in the condition of the core temperature \( T_c = (1 - 2) \times 10^8 \) K (Ayasli & Joss 1982, Hanawa & Fujimoto 1984). For these
Table 12: Non-equilibrium processes for the inner crust. Adopted from Haensel and Zdunik (1990).

<table>
<thead>
<tr>
<th>$P$ (dyn cm$^{-2}$)</th>
<th>Reaction</th>
<th>deposited heat (MeV/nucleon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.980 \times 10^{30}$</td>
<td>$^{52}\text{S} \rightarrow ^{46}\text{Si}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.09</td>
</tr>
<tr>
<td>$2.253 \times 10^{30}$</td>
<td>$^{46}\text{Si} \rightarrow ^{40}\text{Mg}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.10</td>
</tr>
<tr>
<td>$2.637 \times 10^{30}$</td>
<td>$^{40}\text{Mg} \rightarrow ^{34}\text{Ne}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$; $^{34}\text{Ne}+^{34}\text{Ne} \rightarrow ^{68}\text{Ca}$</td>
<td>0.47</td>
</tr>
<tr>
<td>$2.771 \times 10^{30}$</td>
<td>$^{68}\text{Ca} \rightarrow ^{62}\text{Ar}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$3.216 \times 10^{30}$</td>
<td>$^{62}\text{Si} \rightarrow ^{56}\text{S}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.05</td>
</tr>
<tr>
<td>$3.825 \times 10^{30}$</td>
<td>$^{56}\text{S} \rightarrow ^{50}\text{Si}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.06</td>
</tr>
<tr>
<td>$4.699 \times 10^{30}$</td>
<td>$^{50}\text{Si} \rightarrow ^{44}\text{Mg}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.07</td>
</tr>
<tr>
<td>$6.043 \times 10^{30}$</td>
<td>$^{44}\text{Mg} \rightarrow ^{36}\text{Ne}+8\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$; $^{36}\text{Ne}+^{36}\text{Ne} \rightarrow ^{72}\text{Ca}$; $^{72}\text{Ca} \rightarrow ^{66}\text{Ar}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.28</td>
</tr>
<tr>
<td>$7.233 \times 10^{30}$</td>
<td>$^{66}\text{Ar} \rightarrow ^{60}\text{S}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.02</td>
</tr>
<tr>
<td>$9.238 \times 10^{30}$</td>
<td>$^{60}\text{S} \rightarrow ^{54}\text{Si}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.02</td>
</tr>
<tr>
<td>$1.228 \times 10^{31}$</td>
<td>$^{54}\text{Si} \rightarrow ^{48}\text{Mg}+6\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.03</td>
</tr>
<tr>
<td>$1.602 \times 10^{31}$</td>
<td>$^{48}\text{Mg}+^{48}\text{Mg} \rightarrow ^{96}\text{Cr}$</td>
<td>0.11</td>
</tr>
<tr>
<td>$1.613 \times 10^{31}$</td>
<td>$^{96}\text{Cr} \rightarrow ^{88}\text{Ti}+8\text{n} \ 	ext{-2e}^{-}+2\nu_{e}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

accretion rates, only a few percent of the hydrogen/helium burning luminosity diffuses inward and heats the core (Fujimoto et al. 1984, 1987). There is an additional furnace in the inner crust (region above neutron drip, $\rho \geq 5 \times 10^{11}$ g cm$^{-3}$), where the compression of matter by accretion induces electron capture, neutron emissions, and pycnonuclear reactions. These reactions release the energy

$$Q_i = 6.03 \dot{M}_{-10} \frac{q_i}{1 \text{ MeV}} 10^{33} \text{ ergs g}^{-1},$$  \hspace{1cm} (E1)

where $i$ is the number of the reaction, $\dot{M}_{-10}$ mass accretion rate in units of $10^{-10} M_{\odot}$ yr$^{-1}$, and $q_i$ the effective heat energy per nucleon in the $i$-th reaction (Haensel et al. 1990). Haensel and Zdunik (1990) proposed the non-equilibrium processes and estimated $q_i$ in (E1). Tables 12 and 13 show the relations between $P$, $q_i$, and the reaction processes. Non-equilibrium processes generated the heat for the pressure $P$ is indicated in the first column. The third
Table 13: Same as Table 12 but for the outer crust.

<table>
<thead>
<tr>
<th>$P$ (dyn cm$^{-2}$)</th>
<th>Reaction</th>
<th>deposited heat (MeV/nucleon)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.235 \times 10^{26}$</td>
<td>$^{56}\text{Fe} \rightarrow ^{56}\text{Cr} - 2e^- + 2\nu_e$</td>
<td>0.01</td>
</tr>
<tr>
<td>$9.569 \times 10^{27}$</td>
<td>$^{56}\text{Cr} \rightarrow ^{56}\text{Ti} - 2e^- + 2\nu_e$</td>
<td>0.01</td>
</tr>
<tr>
<td>$1.152 \times 10^{29}$</td>
<td>$^{56}\text{Ti} \rightarrow ^{56}\text{Ca} - 2e^- + 2\nu_e$</td>
<td>0.01</td>
</tr>
<tr>
<td>$4.747 \times 10^{29}$</td>
<td>$^{56}\text{Ca} \rightarrow ^{56}\text{Ar} - 2e^- + 2\nu_e$</td>
<td>0.01</td>
</tr>
<tr>
<td>$1.361 \times 10^{30}$</td>
<td>$^{56}\text{Ar} \rightarrow ^{52}\text{S} + 4n - 2e^- + 2\nu_e$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

column corresponds to each $q_i$. These reactions transport most of their heat to the core and therefore play an important role in maintaining the interior thermal balance in a constantly accreting neutron star (Miralda-Escudé et al. 1990, Bildsten & Brown 1997, Brown & Bildsten 1998).

**F. Watershed of Heat Flow**

We discuss how thermal properties of ‘watershed’ come about and how they depend on relevant parameters.

First, we define the “potential luminosity” $\lambda$ by

$$\lambda \equiv \frac{4\pi cGM_\odot}{\kappa} \frac{aT^4}{3P}.$$  \hspace{1cm} (F1)

Here $a$ is the radiation density constant. This potential luminosity controls the heat flow rate $L_{phr}$ at the shell of radius $r$ according to the equation

$$L_{phr} = \lambda \frac{M_q}{M_\odot} 4 \left( \frac{d\ln T}{d\ln P} \right) [G\mathcal{H}\mathcal{V}].$$  \hspace{1cm} (F2)

Here $M_q$ is the rest mass included inside the “Lagrange” coordinate $q (= M_r/M)$, which is set to be an outermost meshpoint ($q = 1 - 4.092 \times 10^{-20}$ for our evolutionary code). The quantities in the square brackets represent the effects of general relativity (see Thorne 1977): the gravitational acceleration factor $G$, the enthalpy correction factor $\mathcal{H}$, and the
volume correction factor $\mathcal{V}$ (B8). The value of $\lambda$ is larger for higher temperature because of a stronger radiation field. The sign against the pressure variation changes; for lower pressure, where heat is transported by radiation, $\lambda$ decreases with increasing pressure, while for higher pressure, where the conduction by degenerate electrons is dominant, it increases with increasing pressure. These two regions are separated by the locus of points at which $\kappa_P + 1 = 0$ as explained below.

The heat flow in the envelope changes its nature according to the sign of $\kappa_P + 1$. We may put $q = 1$ and neglect the variations of the terms due to the general relativistic effects in (F2), since we are here concerned with the outer layers above which there is very little mass. Then, under the assumption of constant luminosity, the equation of radiative equilibrium can be integrated (Schwarzschild 1958; Hayashi et al. 1962), and it follows that the temperature gradient $\nabla (\equiv d\ln T/d\ln P)$ depends on the pressure $P$,

$$\nabla = \left(1 + \frac{\kappa_P}{4 - \kappa_T}\right) \left[1 + \left(\frac{\kappa_P}{4 - \kappa_T} / \nabla_0 - 1\right) \left(\frac{P_0}{P}\right)^{\kappa_P+1}\right]^{-1},$$

where $\kappa_P$ and $\kappa_T$ are the derivatives of the opacity with respect to pressure and temperature ($\kappa_P \equiv \partial \ln \kappa / \partial \ln P |_T$ and $\kappa_T \equiv \partial \ln \kappa / \partial \ln T |_P$), and $\nabla_0$ is the temperature gradient at an arbitrary point where $P = P_0$. This equation gives a physically meaningful result as long as

$$0 \leq \nabla_0 / \left(\frac{1 + \kappa_P}{4 - \kappa_T}\right) \leq 1.$$  

Otherwise, it leads to an unphysical situation: the temperature becomes negative beyond the point where the pressure is

$$P = P_0 \left[1 - \left(\frac{1 + \kappa_P}{4 - \kappa_T}\right) / \nabla_0\right]^{-1/(\kappa_P+1)}.$$  

Thus, from (F3) the heat flow is naturally directed outward in the radiative region where $\kappa_P + 1 > 0$ and inward in the conductive region where $\kappa_P + 1 < 0$. In this reason, we call the boundary that divides the two region as the “watershed” of the heat flow in the envelope.
G. Properties of Bursts occurred in the Early Phase of Accretion

In this appendix, we examine the properties of 12 bursts during the early stages of “period 1” given in §7.3.3. Woosley et al. (2003) calculated successive 12 bursts for $Z = Z_\odot$ and $\dot{M} = 1.75 \times 10^{-9} M_\odot$ yr$^{-1}$. We cite also their results and compare to ours.

Figures 101 and 102 show the light curves from the beginning of accretion to $6 \times 10^5$ s and $5.7 \times 10^4$ s, respectively. After $2.1 \times 10^5$ s, the recurrence time in bursts becomes longer than that of bursts occurred before the time shown in Fig. 101. This is because the bottom of the accreted matter is $\log P = 23.8$, which depth does not reach the ignition curve of the $^{12}\text{C} + ^{12}\text{C}$ reaction for $\log T = 8.3 - 8.4$. However the pressure range exceeds the ignition curve of the $3\alpha$ reaction ($\log P = 22.6$ for $\log T = 8.3$). Since there exists significant amount of produced matters in the deeper than the region where hydrogen/helium burnings occurred (see Fig. 103), the heat transported to the core by them lengthens the recurrence intervals of bursts. Figure 103 shows the composition distribution at the time of the arrow in Fig. 101. Though the convection occurs at $\log P = 22.1 - 22.8$ after the end of the burst, effects on the luminosity is negligible. We have recognized that “long interval 1” in Fig. 35 is a remarkable case for the delay to the next burst. In Fig. 102, the marks of ♯a-l correspond with the bursts from the beginning of the accretion. These bursts occurred regularly with the similar burst properties except for the first burst ♯a.

Comparison between our results and those of Woosley et al. (2003) are presented in Tables 14 and 15. The values from the second column to the fourth one in Table 14 are smaller than those in Table 15, but those of the fifth column are larger. There are crucial differences to affect the bursts such as $\dot{M}$, the criterion of the convection, the size of the nuclear reaction network and the boundary condition for the numerical treatment of the neutron star. Especially, the values of the first column $\Delta t$ depend on the $\dot{M}$. We can estimate $\Delta t$ simply. From eqs. (C2) and (C8),

$$P = g_s \frac{\Delta M}{4 \pi R^2}.$$  \hspace{1cm} (G1)
We substitute $\Delta M = \dot{M} \times \Delta t$ for (G1) and assume the same pressure of the ignition for both calculations. Then, we obtain that $\Delta t_{\text{Woosley}} = 2.76 \Delta t_{\text{our}}$. This estimation is consistent with the values of $\Delta t$ in Tables 14 and 15. Our computation with $\dot{M} = 5 \times 10^{-9} M_\odot \text{ yr}^{-1}$ needs more time for rising. For reference, the properties of the bursts ♯2-5 and *β ($\dot{M} = 1 \times 10^{-9} M_\odot \text{ yr}^{-1}$) are also presented in Table 16.

We conclude that though it needs a large nuclear reaction network to know the detailed properties of a single burst, it needs a more realistic treatment of the neutron star to know the recurrence time, composition distribution and heat transport.

Fig. 101.— Light curves from the beginning of accretion to $t = 6 \times 10^5$ s. The left upper mark of ♯a-l shows the light curve in Fig. 102.
Fig. 102.— Light curves from the beginning to \( t = 5.7 \times 10^4 \) s. Marks of ♯a-l indicate each burst from the beginning of accretion (\( t = 0 \)).

Table 14: Properties of the bursts ♯a-l in Fig. 102.

<table>
<thead>
<tr>
<th>Burst</th>
<th>♯</th>
<th>( \Delta t^\ast )</th>
<th>( L_{\text{max}} )</th>
<th>( \tau_{1/2}^\ast )</th>
<th>( \tau_{\text{rise}}^\ast )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>♯a</td>
<td>1.77</td>
<td>0.91</td>
<td>15</td>
<td>0.84</td>
</tr>
<tr>
<td>b</td>
<td>♯b</td>
<td>1.46</td>
<td>0.81</td>
<td>12</td>
<td>0.97</td>
</tr>
<tr>
<td>c</td>
<td>♯c</td>
<td>1.02</td>
<td>0.81</td>
<td>11</td>
<td>0.90</td>
</tr>
<tr>
<td>d</td>
<td>♯d</td>
<td>1.17</td>
<td>0.85</td>
<td>11</td>
<td>0.95</td>
</tr>
<tr>
<td>e</td>
<td>♯e</td>
<td>1.11</td>
<td>0.82</td>
<td>11</td>
<td>0.92</td>
</tr>
<tr>
<td>f</td>
<td>♯f</td>
<td>1.10</td>
<td>0.76</td>
<td>12</td>
<td>0.93</td>
</tr>
<tr>
<td>g</td>
<td>♯g</td>
<td>1.02</td>
<td>0.86</td>
<td>11</td>
<td>0.93</td>
</tr>
<tr>
<td>h</td>
<td>♯h</td>
<td>1.00</td>
<td>0.79</td>
<td>12</td>
<td>0.98</td>
</tr>
<tr>
<td>i</td>
<td>♯i</td>
<td>1.08</td>
<td>0.84</td>
<td>12</td>
<td>0.98</td>
</tr>
<tr>
<td>j</td>
<td>♯j</td>
<td>1.06</td>
<td>0.79</td>
<td>11</td>
<td>0.95</td>
</tr>
<tr>
<td>k</td>
<td>♯k</td>
<td>1.04</td>
<td>0.77</td>
<td>12</td>
<td>0.94</td>
</tr>
<tr>
<td>l</td>
<td>♯l</td>
<td>0.99</td>
<td>0.79</td>
<td>12</td>
<td>0.98</td>
</tr>
</tbody>
</table>

\(*^1\) Time measured from the last burst (♯b-l). For the first burst (♯a), time is measured from the start of accretion.

\(*^2\) Time for the burst to decay from \( L_{\text{max}} \) to 50% of it.

\(*^3\) Time for the burst to rise from 10% to 50% of the \( L_{\text{max}} \).
Fig. 103.— composition distribution at $2.1 \times 10^5$ s in Fig. 101. The composition at $\log P \geq 24$ is assumed to be $^{56}\text{Ni}$. 
Table 15: Same as Table 14 but for $Z = Z_{\odot}$ and $\dot{M} = 1.75 \times 10^{-9} M_{\odot}$ yr$^{-1}$ in Woosley et al. (2003).

<table>
<thead>
<tr>
<th>Burst</th>
<th>$\Delta t$ (h)</th>
<th>$L_{\text{max}}$ ($10^{38}$ erg s$^{-1}$)</th>
<th>$\frac{1}{2} \tau_{\text{rise}}$ (s)</th>
<th>$\tau_{\text{rise}}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.47</td>
<td>1.91</td>
<td>15</td>
<td>0.66</td>
</tr>
<tr>
<td>2</td>
<td>2.66</td>
<td>1.51</td>
<td>16</td>
<td>0.55</td>
</tr>
<tr>
<td>3</td>
<td>2.72</td>
<td>1.58</td>
<td>17</td>
<td>0.56</td>
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<tr>
<td>4</td>
<td>2.69</td>
<td>1.51</td>
<td>18</td>
<td>0.59</td>
</tr>
<tr>
<td>5</td>
<td>2.65</td>
<td>1.63</td>
<td>15</td>
<td>0.54</td>
</tr>
<tr>
<td>6</td>
<td>2.74</td>
<td>1.54</td>
<td>17</td>
<td>0.58</td>
</tr>
<tr>
<td>7</td>
<td>2.65</td>
<td>1.52</td>
<td>16</td>
<td>0.59</td>
</tr>
<tr>
<td>8</td>
<td>2.68</td>
<td>1.50</td>
<td>17</td>
<td>0.57</td>
</tr>
<tr>
<td>9</td>
<td>2.65</td>
<td>1.57</td>
<td>17</td>
<td>0.57</td>
</tr>
<tr>
<td>10</td>
<td>2.68</td>
<td>1.55</td>
<td>16</td>
<td>0.55</td>
</tr>
<tr>
<td>11</td>
<td>2.68</td>
<td>1.65</td>
<td>17</td>
<td>0.51</td>
</tr>
<tr>
<td>12</td>
<td>2.73</td>
<td>1.66</td>
<td>16</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 16: Same as Table 14 but for the bursts $\sharp$2-5 and $\ast \beta$ (see Figs. 36 and 45 and Table 11).

<table>
<thead>
<tr>
<th>Burst</th>
<th>$\Delta t$ (h)</th>
<th>$L_{\text{max}}$ ($10^{38}$ erg s$^{-1}$)</th>
<th>$\frac{1}{2} \tau_{\text{rise}}$ (s)</th>
<th>$\tau_{\text{rise}}$ (s)</th>
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</thead>
<tbody>
<tr>
<td>$\sharp$2</td>
<td>185</td>
<td>0.82</td>
<td>12</td>
<td>0.84</td>
</tr>
<tr>
<td>3</td>
<td>898</td>
<td>0.82</td>
<td>12</td>
<td>0.84</td>
</tr>
<tr>
<td>4</td>
<td>1617</td>
<td>0.77</td>
<td>13</td>
<td>0.84</td>
</tr>
<tr>
<td>5</td>
<td>31</td>
<td>0.78</td>
<td>13</td>
<td>0.64</td>
</tr>
<tr>
<td>$\ast \beta$</td>
<td>98</td>
<td>0.68</td>
<td>10</td>
<td>1.25</td>
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</tbody>
</table>
Acknowledgments

The author is grateful to his adviser, Professor M. Hashimoto for his valuable advice, continuous encouragement, useful discussion and critical reading of this manuscript. It was indispensable for the completion of this thesis. The author thanks Professor K. Arai for reading the manuscript and the useful advice, and Professor K. Sagara for the fruitful discussions and the useful advice. She wishes to thank Professor M. Y. Fujimoto for useful discussions and the useful advice regarding not only the numerical results but also the coding of the program used in this thesis. She is also grateful to Dr. H. Yamaoka and Dr. T. Yoshida for his useful advice. The members of the Hashimoto group, Mr. Naito, Miss Yamauchi, Mr. Gamow, Mr. Yasutake, Mr. Kuroda, Mr. S. Nishimura, Mr. Nakashima, Mr. Nakayama and Mr. N. Nishimura gave me useful advice. Especially, the shell flash model and FNRN were owed to Dr. O. Koike very much. Finally the author thanks her families and friends for their affection.
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